6. (15 points) For each of the following statements, circle T if the statement is always true, and otherwise circle F. No explanations are required.

(a) The Taylor series for \( \sin(x) \) about \( x = 1 \) is \( (x - 1) - \frac{(x - 1)^3}{3!} + \frac{(x - 1)^5}{5!} - \cdots \)

T    F

(b) If Euler’s method with 10 steps is used to approximate the solution to the initial value problem \( \frac{dy}{dx} = -y, \ y(0) = 1 \) at \( x = 1 \), then the approximation will be an overestimate for the exact solution.

T    F

(c) Let \( f \) be a continuous, positive, decreasing function defined for \( x \geq 1 \) such that \( \int_1^\infty f(x) \, dx \) converges. If \( a_n = f(n) \), then \( \sum_{n=1}^\infty a_n = \int_1^\infty f(x) \, dx \).

T    F

(d) The system of differential equations,

\[
\begin{align*}
\frac{1}{x} \frac{dx}{dt} &= y - 1, \\
\frac{1}{y} \frac{dy}{dt} &= x - 1,
\end{align*}
\]

models the interaction of two populations involved in a predator-prey relationship.

T    F

(e) The relative growth rate of the population in the \textit{logistic model for population growth} is a linear function of the population.

T    F