1. (4 points) Circle the differential equation whose slope field is shown in the figure.

A. 
$$\frac{dy}{dx} = \sin x$$

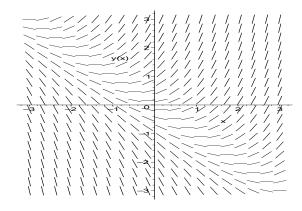
$$B. \quad \frac{dy}{dx} = -y$$

A. 
$$\frac{dy}{dx} = \sin x$$
 B.  $\frac{dy}{dx} = -y$  C.  $\frac{dy}{dx} = x^2 + y^2$ 

D. 
$$\frac{dy}{dx} = x + y$$
 E.  $\frac{dy}{dx} = x - 2y$  F.  $\frac{dy}{dx} = \sin(x + y)$ 

$$E. \quad \frac{dy}{dx} = x - 2y$$

F. 
$$\frac{dy}{dx} = \sin(x+y)$$



2. (6 points) The function f is a continuous function, some of whose values are given in the following table.

x	0	1	2	3	4	5	6
f(x)	8	6	3	-2	0	1	2

For the function F defined by  $F(x) = \int_0^x f(t)e^{-t} dt$ , what is F'(2)?

$$F'(2) = \underline{f(2)e^{-2}} = 3e^{-2} \approx .4060058496.$$

3. (6 points) Does the infinite series  $\sum_{n=1}^{\infty} ne^{-n^2}$  converge or diverge? (Show your work.)

Apply the integral test. The function  $f(x) = xe^{-x^2}$  is a positive, decreasing function of x for  $x \ge 1$  with  $f(n) = ne^{-n^2}$ . Also, since  $\frac{d}{dx}e^{-x^2} = -2xe^{-x^2}$ , we have by the fundamental theorem of calculus that

$$\int_{1}^{\infty} x e^{-x^{2}} dx = \lim_{b \to \infty} \frac{1}{2} (e^{-1} - e^{-b^{2}}) = \frac{1}{2e} < \infty$$

so  $\sum_{i=1}^{\infty} ne^{-n^2}$  converges by the integral test.