

6. (15 points) For each of the following statements, circle **T** if the statement is always true, and otherwise circle **F**. No explanations are required.

(a) The Taylor series for $\sin(x)$ about $x = 1$ is $(x - 1) - \frac{(x - 1)^3}{3!} + \frac{(x - 1)^5}{5!} - \dots$

T

F

(b) If Euler's method with 10 steps is used to approximate the solution to the initial value problem $\frac{dy}{dx} = -y$, $y(0) = 1$ at $x = 1$, then the approximation will be an overestimate for the exact solution.

T

F

(c) Let f be a continuous, positive, decreasing function defined for $x \geq 1$ such that $\int_1^{\infty} f(x) dx$ converges. If $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n = \int_1^{\infty} f(x) dx$.

T

F

(d) The system of differential equations,

$$\begin{aligned} \frac{1}{x} \frac{dx}{dt} &= y - 1, \\ \frac{1}{y} \frac{dy}{dt} &= x - 1, \end{aligned}$$

models the interaction of two populations involved in a predator-prey relationship.

T

F

(e) The relative growth rate of the population in the *logistic model for population growth* is a linear function of the population.

T

F