8. (10 points) The *electric potential* is a quantity of great importance in electrostatics. The electric potential $V(R)$ at a distance R along the axis perpendicular to the center of a charged disk with radius 1 is given by

$$V(R) = C\left(\sqrt{R^2+1} - R\right)$$

where C is a constant that depends on the choice of units that are being used.

(a) Show that for large numbers R,

$$V(R) \approx \frac{C}{2R}.$$

(Hint: $\sqrt{R^2+1} = R\sqrt{1+\frac{1}{R^2}}$ and remember that R is large.)

As the hint says, $\sqrt{R^2+1} = R\sqrt{1+\frac{1}{R^2}} = R(1+\frac{1}{R^2})^{\frac{1}{2}}$.
Recall (or calculate) that for $-1 < x < 1$, *we have* $(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \ldots$
Thus for R *large (enough so that* $-1 < \frac{1}{R^2} < 1$*) we have*
$(1+\frac{1}{R^2})^{\frac{1}{2}} = 1 + \frac{1}{2}\frac{1}{R^2} - \frac{1}{8}(\frac{1}{R^2})^2 + \frac{1}{16}(\frac{1}{R^2})^3 - \ldots = 1 + \frac{1}{2}\frac{1}{R^2} - \frac{1}{8}\frac{1}{R^4} + \frac{1}{16}\frac{1}{R^6} - \ldots$
So $R(1+\frac{1}{R^2})^{\frac{1}{2}} = R + \frac{1}{2}\frac{1}{R} - \frac{1}{8}\frac{1}{R^3} + \frac{1}{16}\frac{1}{R^5} - \ldots$
Therefore $R(1+\frac{1}{R^2})^{\frac{1}{2}} - R = \frac{1}{2}\frac{1}{R} - \frac{1}{8}\frac{1}{R^3} + \frac{1}{16}\frac{1}{R^5} - \ldots$
So $C(R(1+\frac{1}{R^2})^{\frac{1}{2}} - R) = \frac{C}{2}\frac{1}{R} - \frac{C}{8}\frac{1}{R^3} + \frac{C}{16}\frac{1}{R^5} - \ldots$
And so we have that for large numbers R, *we can approximate* $C(R(1+\frac{1}{R^2})^{\frac{1}{2}} - R)$ *by* $\frac{C}{2}\frac{1}{R}$.

(b) Approximately how large should R be in order that the error in the approximation of $V(R)$ by $C/2R$ is less than 4% of $V(R)$?

For large R, *the error in the approximation of* $V(R)$ *by* $C/2R$ *is approximately* $\frac{-C}{8R^3}$.
So, we want to solve $\frac{C}{8R^3} < .04V(R)$ *for* R.
This is approximately the same as solving $\frac{C}{8R^3} < .04\frac{C}{2R}$ *for* R.
That is, $1/4 < .04R^2$ *or* $R > 1/.4 = 2.5$. *Thus*

R should approximately be greater than 2.5