2. [18 points] For each of the following series, write whether the series "Converges" or "Diverges" on the space provided next to the series. Support your answer by stating the test(s) you used to prove convergence or divergence, and show complete work and justification.

a. [6 points] $\sum_{n=2}^{\infty} \frac{\sqrt{n^2+1}}{n^2-1}$ **Diverges**

Solution: We note that $\frac{1}{n^2-1} > \frac{1}{n^2}$ and $\sqrt{n^2+1} > n$ for $n \geq 2$, so $\frac{\sqrt{n^2+1}}{n^2-1} > \frac{1}{n}$. We know $\sum_{n=2}^{\infty} \frac{1}{n}$ diverges by the p-test, so by the comparison test, the series diverges.

b. [6 points] $\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(2n)!}$ **Converges**

Solution: We use the ratio test to get

$$\lim_{n\to\infty} \frac{(2n)!(n+1)!(n+2)!}{(2(n+1))!(n+1)!n!}.$$

After some cancellation we get

$$\lim_{n\to\infty} \frac{(n+2)(n+1)}{(2n+1)(2n+2)} = 1/4.$$

Thus it converges.

c. [6 points] $\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2-3}$ **Converges**

Solution: We will show that this is absolutely convergent, and hence convergent. Additionally, since $|\sin(n)| \leq 1$ we can bound the sum of the absolute values by

$$\sum_{n=2}^{\infty} \frac{1}{n^2-3}.$$

Then we apply the limit comparison test with $\sum_{n=2}^{\infty} \frac{1}{n^2}$. Since

$$\lim_{n\to\infty} \frac{n^2-3}{n^2} = 1$$

both series converge or both diverge. Since $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges by p-test, they both converge.