

3. [12 points] For each of the following series, determine the interval of convergence and write it on the space provided to the right of the series. Be sure to show all appropriate work to justify your answer.

a. [6 points] $\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{n}$ _____ $1 < x \leq 3$ _____

Solution: By using the ratio test we see that

$$\lim_{n \rightarrow \infty} \frac{|x-2|^{n+1}(n)}{|x-2|^n(n+1)} = |x-2|.$$

So the center is at $x = 2$ and the radius is 1. We need to check the endpoints $x = 1$ and $x = 3$. We see it converges when $x = 3$ by alternating series, and diverges at $x = 1$ by the p-test.

b. [6 points] $\sum_{n=1}^{\infty} \frac{n!x^n}{n^{10}}$ _____ $x = 0$ _____

Solution: By using the ratio test we get

$$\lim_{n \rightarrow \infty} \frac{(n+1)!x^{n+1}n^{10}}{n!x^n(n+1)^{10}} = \infty.$$

So it has radius of convergence 0 and converges only at the center which is $x = 0$.