4. [8 points] Consider a solid whose base is contained between the curves $y = e^x$, $y = 1$, and $x = 3$. Cross-sectional slices perpendicular to the x-axis are rectangles, having length contained in the base region mentioned above and height determined by $g(x) = x^2$. Determine the exact volume of this solid.

Solution: The slice has volume $x^2(e^x - 1)\Delta x$. Summing the slices and letting Δx go to 0, we have

$$\begin{aligned}
\text{Volume} &= \int_0^3 x^2(e^x - 1)dx \\
&= \int_0^3 x^2 e^x dx - \int_0^3 x^2 dx \\
&= (x^2e^x|_0^3 - \int_0^3 2xe^x dx) - \frac{1}{3}x^3|_0^3 \\
&= (x^2e^x|_0^3 - (2xe^x|_0^3 - \int_0^3 2e^x dx)) - \frac{1}{3}x^3|_0^3 \\
&= (x^2e^x - 2xe^x + 2e^x - \frac{1}{3}x^3)|_0^3 \\
&= 9e^3 - 6e^3 + 2e^3 - 9 - 2 \\
&= 5e^3 - 11
\end{aligned}$$