**6**. [10 points] Consider the function $f(x) = \ln(1+x)$ and its Taylor series about $x = 0$.

**a**. [4 points] Determine the first four non-zero terms of the Taylor series for $f(x) = \ln(1+x)$ about $x = 0$. Be sure to show enough work to support your answer.

*Solution:*

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + ...$$

**b**. [4 points] Find the first three non-zero terms of the Taylor series for $g(x) = \ln\left(\frac{1+x}{1-x}\right)$ about $x = 0$. Be sure to show enough work to support your answer. *(Hint: You may find it helpful to utilize properties of logarithms.)*

*Solution:* Using our answer from part (a), we have

$$\ln(1-x) = (-x) - \frac{1}{2}(-x)^2 + \frac{1}{3}(-x)^3 - \frac{1}{4}(-x)^4 + ... = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4 - ...$$

Since $g(x) = \ln(1+x) - \ln(1-x)$, we have

$$g(x) = 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + ...$$

**c**. [2 points] Find the exact value of the sum of the series

$$2\left(\frac{3}{4}\right) + \frac{2}{3}\left(\frac{3}{4}\right)^3 + \frac{2}{5}\left(\frac{3}{4}\right)^5 + ... \, .$$

*Solution:* Using our answer for $g(x)$ is part (b), we see this is the sum for the Taylor series evaluated at $x = \frac{3}{4}$, so the sum of this series is $g\left(\frac{3}{4}\right) = \ln(7)$.