1. [10 points] For each question, circle if the statement is always true or false. **No justification is necessary**.

a. [2 points] If $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n$ and $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges.

True [False]

Solution: Example: If $a_n = \frac{1}{n^2}$ and $b_n = \frac{1}{n}$, then $\sum_{n=1}^{\infty} a_n$ converges and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = 0$, but $\sum_{n=1}^{\infty} b_n$ diverges.

b. [2 points] If $F(x) = \int_{1-x}^{x^3} e^{-t^4} dt$ then $F(x)$ is increasing.

[True] False

Solution: $F'(x) = 3x^2 e^{-x^{12}} + e^{-(1-x)^4} > 0$ then $F(x)$ is increasing.

c. [2 points]

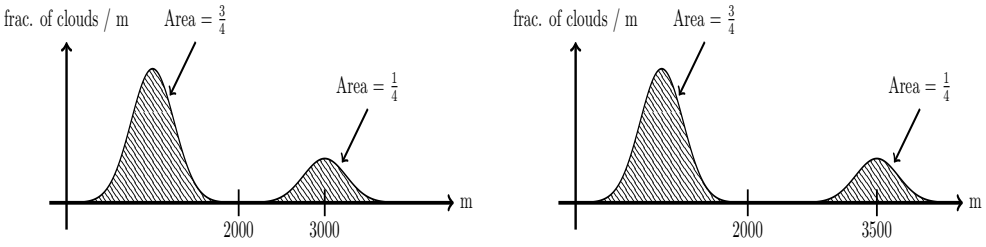
$$\ln(2.5) = 1.5 - \frac{1}{2}(1.5)^2 + \frac{1}{3}(1.5)^3 - \frac{1}{4}(1.5)^4 + \cdots$$

True [False]

Solution: The Taylor series for $\ln(1+x)$ is not valid for $x = 1.5$.

d. [2 points] The left graph is a probability density function for the height of clouds in the sky. A gust of wind causes all clouds higher than 2000 meters to rise an additional 500 meters. The right graph shows the heights of clouds afterwards. The median cloud height after the wind is 500/4 meters higher than the median cloud height before the wind.

True [False]



Solution: Both medians are the same.

e. [2 points] A particle's position is given by the parametric equations $(x(t), y(t))$. If the graphs of $x(t)$ and $y(t)$ are given below, then the particle passes through the origin twice for $0 \le t \le 3$.

True [False]

Solution: The particle passes through the only at $t = 1$.