10. [10 points]

a. [5 points] Determine whether the following series converge or diverge (circle your answer). For each, justify your answer by writing what convergence rule or convergence test you would use to prove your answer. If you use the comparison test or limit comparison test, also write an appropriate comparison function.

1.[2 points]

$$\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n^2}$$

Converge Diverge

Solution: **Diverges** $\lim_{n\to\infty}(-1)^n\frac{2^n}{n^2}\neq 0$.

2.[3 points]

$$\sum_{n=1}^{\infty}\frac{3n-2}{\sqrt{n^5+n^2}}$$

Converge Diverge

Solution: **Converges**
Limit Comparison Test $b_n=\frac{1}{n^{\frac{3}{2}}}$ (or a multiple) and p series $p=\frac{3}{2}>1$.
Comparison Test $b_n=\frac{3}{n^{\frac{3}{2}}}$ and p series $p=\frac{3}{2}>1$.

b. [5 points] Does the following series converge conditionally, absolutely, diverge or is it not possible to decide? Justify.

$$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n(1+\ln(n))}$$

Solution: Is it Absolute Convergent? Using Integral test with $f(x)=\frac{1}{x(1+\ln(x))}$

- $f(x)\geq 0$.
- $f(x)$ decreasing.

$$\sum_{n=1}^{\infty}\frac{1}{n(1+\ln(n))} \text{ behaves as } \int_1^{\infty}\frac{1}{x(1+\ln(x))}dx$$

$$\int_1^{\infty}\frac{1}{x(1+\ln(x))}dx=\lim_{b\to\infty}\int_1^{b}\frac{1}{x(1+\ln(x))}dx=\lim_{b\to\infty}\ln|1+\ln(b)|=\infty$$

Hence $\sum_{n=1}^{\infty}\frac{1}{n(1+\ln(n))}$ is not absolutely convergent.
Is it Conditionally Convergent? Since $a_n=\frac{1}{n(1+\ln(n))}$ decreasing and converges to zero, then by Alternating series test
$\sum_{n=1}^{\infty}(-1)^n\frac{1}{n(1+\ln(n))}$ converges conditionally.