5. [14 points] Years later, after being rescued from the island, you design a machine that will automatically feed wood into a fire at a constant rate of 500 pounds per day. At the same time, as it burns, the weight of the wood pile (in pounds) decreases at a rate (in pounds/day) proportional to the current weight with constant of proportionality $\frac{1}{2}$.

a. [3 points] Let $W(t)$ be the weight of the wood pile t days after you start the machine. Write a differential equation satisfied by $W(t)$.

Solution:

$$W' = -0.5W + 500$$

b. [4 points] Find all equilibrium solutions to the differential equation in part (a). For each equilibrium solution, determine whether it is stable or unstable, and give a practical interpretation of its stability in terms of the weight of the wood pile as $t \to \infty$.

Solution: There is one equilibrium solution, $W = 1000$. This equilibrium solution is stable. This means that if your fire initially contains approximately 1000 pounds of wood, then in the long run the weight of the fire will approach 1000 pounds.

c. [7 points] Solve the differential equation from part (a), assuming that the wood pile weighs 200 pounds when you start the machine.

Solution:

$$\begin{aligned}
\frac{dW}{-0.5W + 500} &= dt \\
-2\ln|-0.5W + 500| &= t + C \\
-0.5W + 500 &= Ae^{-0.5t} \\
W &= 1000 + Ae^{-0.5t} \\
W &= 1000 - 800e^{-0.5t}
\end{aligned}$$