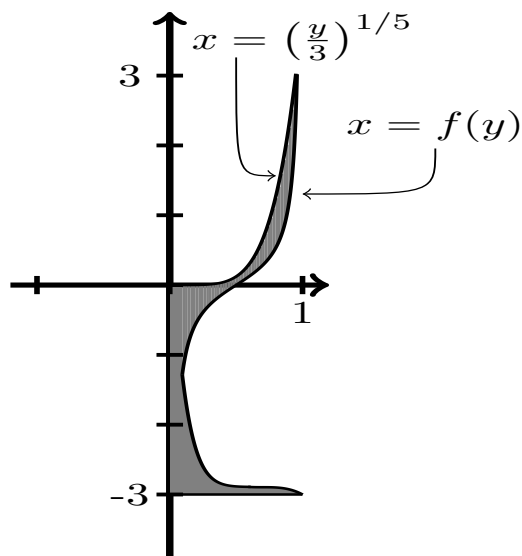**7**. [12 points]

**a**. [5 points] You rotate the region shown about the y-axis to create a drinking glass. Write an expression that represents the volume of material required to construct the drinking glass (your answer may contain $f(y)$).

Solution:

$$\text{Volume} = \int_0^3 \pi \left(f(y)^2 - \left(\frac{y}{3}\right)^{\frac{2}{5}} \right) dy + \int_{-3}^0 \pi f(y)^2 dy.$$

b. [7 points] Consider the vessel shown below. It is filled to a depth of 1 foot of water. Write an integral in terms of y (the distance in ft from the bottom of the vessel) for the work required to pump all the water to the top of the vessel. Water weighs 62.4 lbs/ft^3.
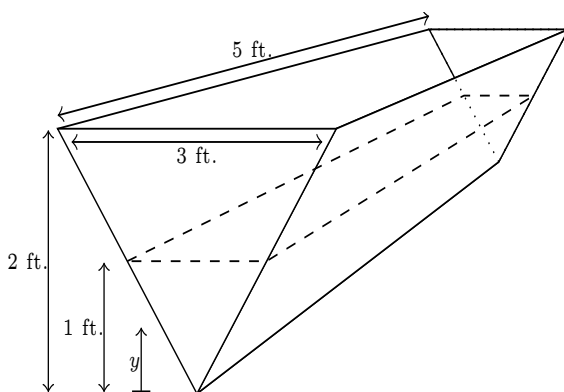
Solution: Using similar triangles:

Volume of a slice$= 5\left(\frac{3}{2}y\right)\Delta y$

Work$=\int_0^1 5\left(\frac{3}{2}y\right)(62.4)(2-y)dy = \int_0^1 468y(2-y)dy.$