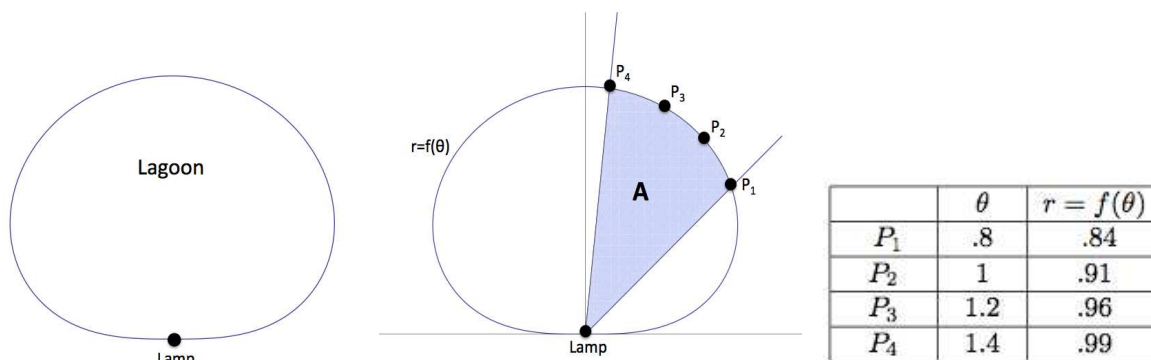


8. [13 points] A lamp is at the border of a small lagoon. The picture below shows the lamp at the origin and the lagoon described by the polar curve $r = f(\theta)$. During the night, the lamp illuminates the shaded region on the lagoon shown below.



- a. [3 points] If A is the shaded area shown above, then $A = \int_a^b F(\theta) d\theta$ where

$F(\theta) =$ _____ $a =$ _____ $b =$ _____.

Solution: $F(\theta) = \frac{1}{2}f(\theta)^2$ $a = .8$ $b = 1.4$

- b. [5 points] Fill the table below with the values of $F(\theta)$. Then approximate the area of the shaded region using Left(3). Write all the terms in your sum.

| | | | | |
|-------------|----|---|-----|-----|
| θ | .8 | 1 | 1.2 | 1.4 |
| $F(\theta)$ | | | | |

Solution:

| | | | | |
|-------------|-------|-------|-------|-----|
| θ | .8 | 1 | 1.2 | 1.4 |
| $F(\theta)$ | .3528 | .4141 | .4608 | .49 |

$$\text{Left}(3) = .2(.3528 + .4141 + .4608) = .2455.$$

- c. [1 point] Is $F(\theta)$ increasing, decreasing or neither?

Solution: Increasing

- d. [1 point] Is $F(\theta)$ concave up, concave down or neither?

Solution: Concave down

- e. [3 points] Which Riemann sums (Left(3), Right(3) and/or Trap(3)) yield an under-estimate of the shaded area?

Solution: Left(3) and Trap(3).