1. [12 points] Indicate whether each of the following statements are true or false by circling the correct answer. **You do not need to justify your answers.**

a. [2 points] The curve defined by the parametric equations $x = 1 - \cos t$ and $y = t - \sin t$ has a vertical tangent line when $t = \pi$.

[✓ True] False

Solution: $x'(\pi) = \sin\pi = 0$ and $y'(\pi) = 1 - \cos(\pi) = 2$, hence the curve has a vertical tangent at $t = \pi$.

b. [2 points] If the sequence a_n converges to 0 and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty}(a_n + b_n)$ converges.

True [✓ False]

Solution: Let $a_n = \frac{1}{n}$ and $b_n = \frac{1}{n^2}$, then $\frac{1}{n}$ converges to 0 and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, but $\sum_{n=1}^{\infty}(\frac{1}{n} + \frac{1}{n^2}) \geq \sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

c. [2 points] The graph of a polar function $r = f(\theta)$ in the (x, y)-plane has a horizontal tangent line at $\theta = a$ if $f'(a) = 0$.

True [✓ False]

Solution: The graph of $r = f(\theta) = \cos\theta$ satisfies $f'(\pi) = -\sin\pi = 0$, but it has no horizontal asymptote at $\theta = \pi$.

d. [2 points] The integral $\int_0^1 \pi x^4 dx$ computes the volume of the solid obtained by rotating the graph of $y = x^2$ around the x axis for $0 \leq x \leq 1$.

[✓ True] False

Solution: $V = \int_0^1 \pi(x^2)^2 dx = \int_0^1 \pi x^4 dx$

e. [2 points] Let $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 1} x^n$ be the Taylor series of $f(x)$ about 0. Then $f(x)$ is concave up at $x = 0$.

[✓ True] False

Solution: The coefficient of x^2 in the Taylor series is $\frac{f''(0)}{2!} = \frac{1}{2^2+1} = \frac{1}{5} > 0$. Hence $f''(0) > 0$ so $f(x)$ is concave up.

f. [2 points] The integral test says that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \int_1^{\infty} \frac{1}{x^2} dx$.

True [✓ False]

Solution: The integral test only says $\sum_{n=1}^{\infty} \frac{1}{n^2}$ behaves as $\int_1^{\infty} \frac{1}{x^2} dx$.