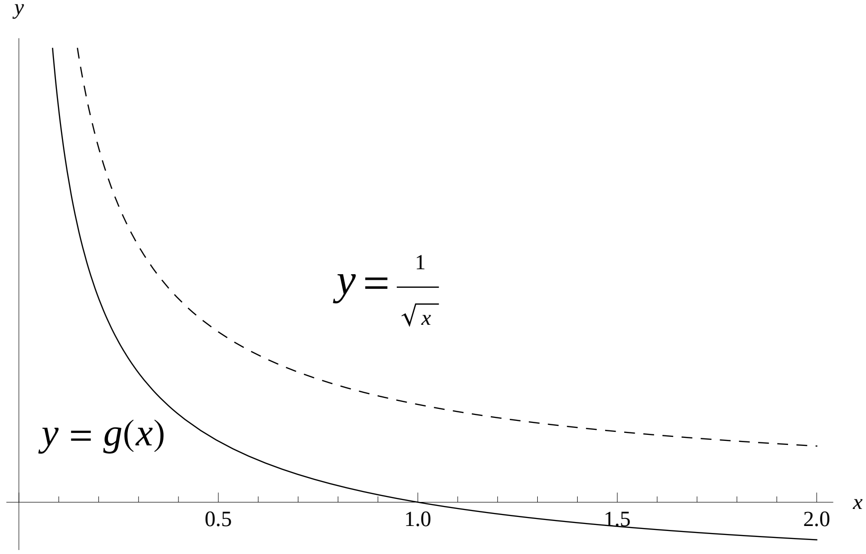
4. [11 points]

a. [2 points] Let $g(x)$ be a continuous function for $x > 0$ and let $G(x)$ be the antiderivative of $g(x)$ with $G(1) = 0$. Write a formula for $G(x)$.

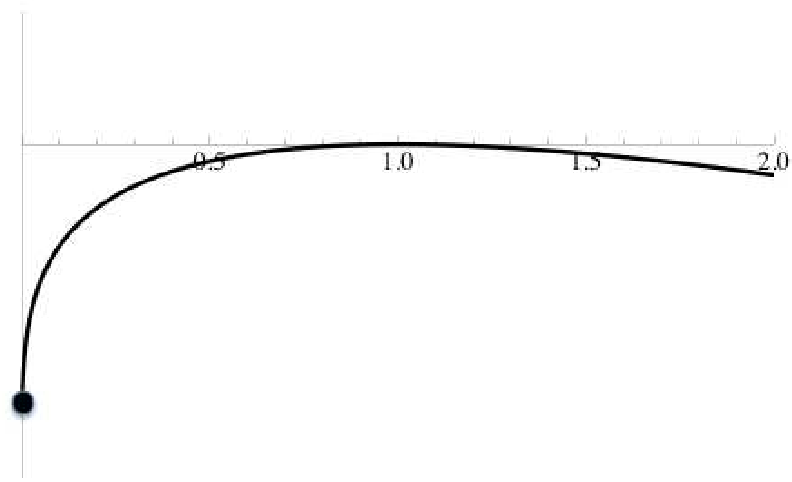
Solution: $G(x) = \int_1^x g(t)dt$

b. [5 points] The graph of $g(x)$ is shown below. The function $g(x)$ has a vertical asymptote at $x = 0$ and $g(x) < \frac{1}{\sqrt{x}}$ for $x > 0$.

Sketch the graph of $G(x)$ for $0 \le x \le 2$. Make sure you indicate where $G(x)$ has asymptotes, local maxima, or local minima, as well as where $G(x)$ is increasing, decreasing, concave up or concave down.



Solution:



c. [4 points] Suppose $h(x)$ and $f(x)$ are continuous functions satisfying

i. $0 < f(x) \leq \dfrac{1}{x^p}$ for $0 < x \leq 1$.

ii. $\dfrac{1}{x^{p+\frac{1}{2}}} \leq h(x) \leq \dfrac{1}{x^p}$ for $x \geq 1$.

Decide whether each of the following expressions converge, diverge or if there is not enough information available to conclude.

Solution:

i. If $p = \dfrac{1}{2}$,

(a) $\lim\limits_{x\to\infty} h(x)$

$\boxed{\textbf{Converges}}$ Diverges Not possible to conclude.

(b) $\displaystyle\int_1^\infty h(x)dx$:

Converges $\boxed{\textbf{Diverges}}$ Not possible to conclude.

ii. If $p = 2$,

(a) $\displaystyle\int_1^\infty h(x)dx$:

$\boxed{\textbf{Converges}}$ Diverges Not possible to conclude.

(b) $\displaystyle\int_0^1 f(x)dx$

Converges Diverges $\boxed{\textbf{Not possible to conclude.}}$