4. [11 points]
   
   a. [2 points] Let \( g(x) \) be a continuous function for \( x > 0 \) and let \( G(x) \) be the antiderivative of \( g(x) \) with \( G(1) = 0 \). Write a formula for \( G(x) \).

   \[ \text{Solution: } G(x) = \int_1^x g(t) \, dt \]

   b. [5 points] The graph of \( g(x) \) is shown below. The function \( g(x) \) has a vertical asymptote at \( x = 0 \) and \( g(x) < \frac{1}{\sqrt{x}} \) for \( x > 0 \).

   Sketch the graph of \( G(x) \) for \( 0 \leq x \leq 2 \). Make sure you indicate where \( G(x) \) has asymptotes, local maxima, or local minima, as well as where \( G(x) \) is increasing, decreasing, concave up or concave down.

   \[ y = \frac{1}{\sqrt{x}} \]
c. [4 points] Suppose $h(x)$ and $f(x)$ are continuous functions satisfying

i. $0 < f(x) \leq \frac{1}{x^p}$ for $0 < x \leq 1$.

ii. $\frac{1}{x^{p+\frac{1}{2}}}$ $\leq h(x) \leq \frac{1}{x^p}$ for $x \geq 1$.

Decide whether each of the following expressions converge, diverge or if there is not enough information available to conclude.

**Solution:**

i. If $p = \frac{1}{2}$,

(a) \( \lim_{x \to \infty} h(x) \)  
\[ \text{Converges} \quad \text{Diverges} \quad \text{Not possible to conclude.} \]

(b) \( \int_1^\infty h(x) \, dx \):
\[ \text{Converges} \quad \text{Diverges} \quad \text{Not possible to conclude.} \]

ii. If $p = 2$,

(a) \( \int_1^\infty h(x) \, dx \):
\[ \text{Converges} \quad \text{Diverges} \quad \text{Not possible to conclude.} \]

(b) \( \int_0^1 f(x) \, dx \):
\[ \text{Converges} \quad \text{Diverges} \quad \text{Not possible to conclude.} \]