6. [9 points] Let $y(t)$ be the number of fish (**in hundreds**) in an artificial lagoon, where t is measured in years. The function $y(t)$ satisfies the following differential equation

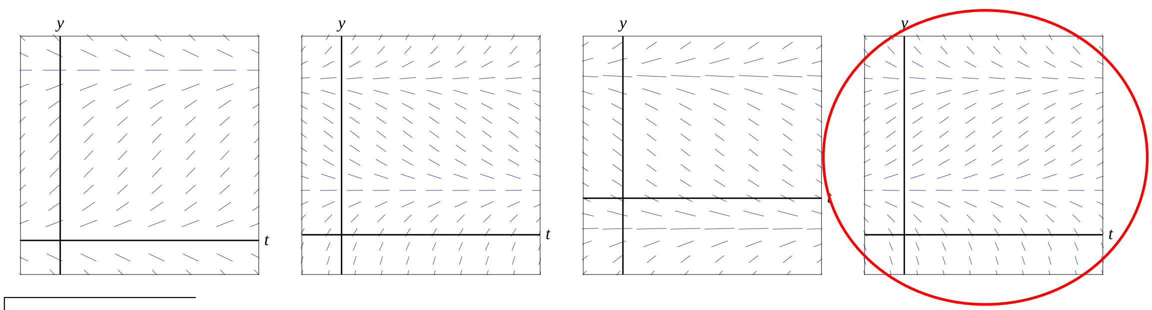
$$\frac{dy}{dt} = y\,(10 - y) - h.$$

where the constant h is the rate at which the fish are harvested from the lagoon.

a. [4 points] Suppose there is no harvesting ($h = 0$). Find the equilibrium solutions of the differential equation. Determine the stability of each equilibrium.

Solution: Equilibrium solutions are found by setting $\frac{dy}{dt} = 0$. So when $h = 0$, we have $y = 0$ and $y = 10$ as equilibrium solutions. Now when $y < 0$ or $y > 10$, we have $\frac{dy}{dt} < 0$. For $0 < y < 10$, we have $\frac{dy}{dt} > 0$. So $y = 0$ is an unstable equilibrium solution and $y = 10$ is a stable equilibrium solutions.

b. [2 points] Suppose the fish are harvested at a rate $h = 9$. Which of the following slope fields may correspond to the differential equation for $y(t)$? Circle your answer.



Solution: The equation is $y' = y(10 - y) - 9$. The equilibrium solutions are $y = 1$ (unstable) and $y = 9$ (stable).

c. [3 points] If (at $t = 0$) there are 200 fish in the lagoon, what is the maximum rate h for harvesting the fish, while still maintaining the fish population in the long run (i.e do not let the fish die out)? (Hint: You do not need to solve the differential equation to answer this question).

Solution: In order for the fish to not die out, we need $\frac{dy}{dt} \geq 0$. That gives us the equation $16 - h \geq 0$, so $h \leq 16$. Therefore 16 is the maximum rate of harvesting for the fish.