

6. [10 points] At a hospital, a patient is given a drug intravenously at a constant rate of r mg/day as part of a new treatment. The patient's body depletes the drug at a rate proportional to the amount of drug present in his body at that time. Let $M(t)$ be the amount of drug (in mg) in the patient's body t days after the treatment started. The function $M(t)$ satisfies the differential equation

$$\frac{dM}{dt} = r - \frac{1}{4}M \quad \text{with} \quad M(0) = 0.$$

- a. [7 points] Find a formula for $M(t)$. Your answer should depend on r .
- b. [1 point] Find all the equilibrium solutions of the differential equation.
- c. [2 points] The treatment's goal is to stabilize in the long run the amount of drug in the patient at a level of 200 mg. At what rate r should the drug be administered?