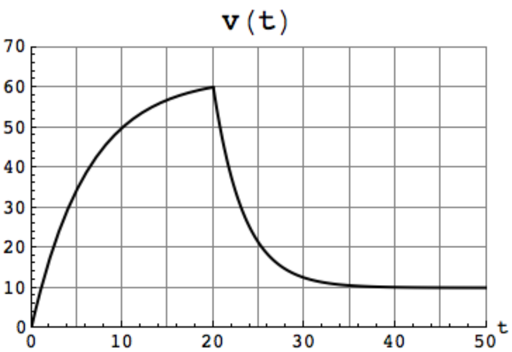
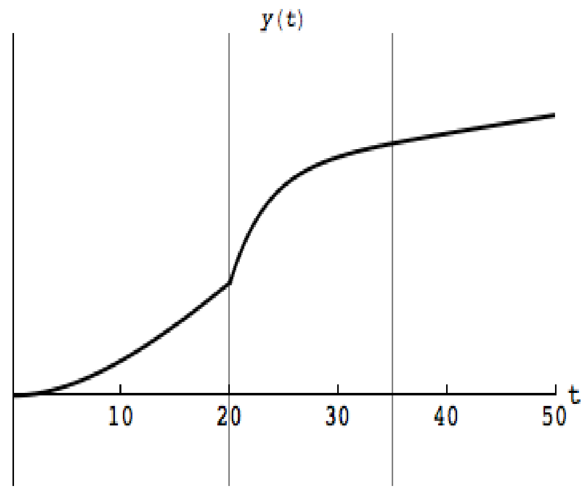
5. [14 points] A skydiver jumps from a plane at a height of 2,000 meters above the ground. After some time in free-fall, he opens his parachute, reducing his speed, and lands safely on the ground.

a. [5 points] The graph of the skydiver's downward velocity $v(t)$ (in meters per second) t seconds after he jumped is shown below.
Sketch the graph of the antiderivative $y(t)$ of $v(t)$ satisfying $y(0) = 0$. Make sure your graph reflects the regions at which the function is increasing, decreasing, concave up or concave down.





It is important to notice that $y'(t)$ exist for all values of t since $y'(t) = v(t)$.

b. [3 points] Write down a right-hand sum with 4 subintervals in order to approximate the **average** downward velocity of the skydiver during the time the skydiver is in free-fall. Show all the terms in your sum.

Solution: The average downward velocity is $\frac{1}{20}\int_0^{20} v(t)dt$. We approximate this as

$$\frac{1}{20}\int_0^{20} v(t)dt \approx \frac{5(35+50+55+60)}{20}$$

c. [2 points] Is your estimate in (**b**) guaranteed to be an underestimate or overestimate of the average velocity of the skydiver, or there is not enough information to decide? Justify.

Solution: It's guaranteed to be an overestimate, because $v(t)$ is increasing throughout $[0, 20]$.

d. [4 points] Find a formula for the height $H(t)$ (in meters) above the ground of the skydiver t seconds after he jumped.

Solution: $H(t) = 2,000 - \int_0^t v(s)ds = 2,000 - y(t)$.