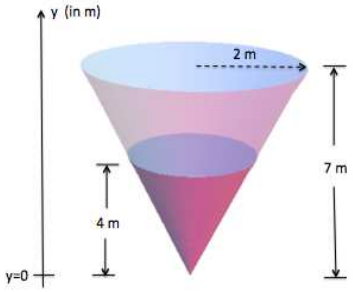
7. [9 points] A tank has the shape of a circular cone. The cone has radius 2 m and height 7 m (as shown below). The tank contains a liquid up to a depth of 4 m. The density of the liquid is $\delta(y) = 1100 - y^2$ kg/m^3, where y measures the distance in meters from the bottom of the tank. Use the value $g = 9.8$ m/s^2 for the acceleration due to gravity.

a. [6 points] Find a definite integral that computes the mass of the liquid in the tank. Show all your work.



Solution: Let $r(y)$ be the radius at height y. By similar triangles, $2/7 = r/y$, so $r = \frac{2}{7}y$. The approximate mass of a thin slice at height y is $\pi(2/7y)^2(1100-y)^2\Delta y$, so the answer is

$$\int_0^4 \pi(2/7y)^2(1100 - y^2)dy.$$

b. [3 points] Find a definite integral that computes the work required to pump the liquid 2 meters above the top of the tank. Show all your work.

Solution: We want to lift each thin slice $(9 - y)$ feet. The work to lift a slice is $9.8(9 - y)\pi(2/7y)^2(1100 - y^2)\Delta y$, so the integral is

$$\int_0^4 9.8(9 - y)\pi(2/7y)^2(1100 - y^2)dy.$$