

1. [10 points]

- a. [4 points] Consider the differential equation  $y' = y^2 + y - 2$ . Find all of the equilibrium solutions of the differential equation and indicate whether they are stable or unstable. Circle your answers.

*Solution:*  $y' = y^2 + y - 2 = (y+2)(y-1)$ . Therefore the equilibrium solutions are  $y = -2$  which is stable and  $y = 1$  which is unstable.

- b. [4 points] Solve the differential equation  $y' = y^2$  with initial condition  $y(0) = 1$ .

*Solution:* Separating variables we have  $\frac{dy}{y^2} = dx$ . Integrating both sides we have  $-\frac{1}{y} = x + c$  therefore  $y = \frac{-1}{x+c}$ . Plugging in the initial condition we must have  $c = -1$ . So  $y = \frac{1}{1-x}$ .

- c. [2 points] Which of the following functions is a solution to the differential equation  $y' = \sin(x) + y$ ? Circle your answer.

$$y = \frac{1}{2}(\sin(x) + \cos(x))$$

$$y = -\frac{1}{2}(\sin(x) - \cos(x))$$

$$y = \frac{1}{2}(\sin(x) - \cos(x))$$

$$y = -\frac{1}{2}(\sin(x) + \cos(x))$$