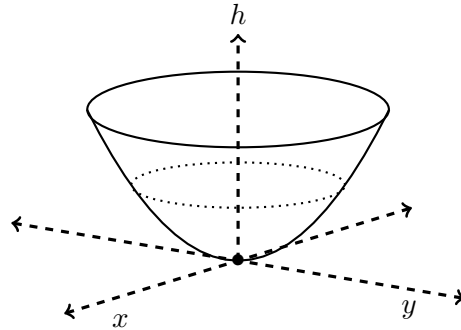


10. [10 points] Martin is having a party to celebrate the beginning of spring and he is serving punch out of a parabolic punch bowl. The bowl is sitting on a table (the  $xy$ -plane) as depicted in the figure below. At a height  $h$  above the table, the cross section of the bowl perpendicular to the  $h$ -axis is a circle with equation,  $h = 4x^2 + 4y^2$ . The punch bowl is 1 meter tall. Assume the units of  $x$ ,  $y$ , and  $h$  are in meters and the density of the punch is  $1200 \text{ kg/m}^3$ . Recall the gravitational constant is  $g = 9.8 \text{ m/s}^2$ .



- a. [5 points] Write an expression for the mass of a slice of punch of thickness  $\Delta h$  meters at a height  $h$  meters above the table.

$$\boxed{\text{Solution: } M = 1200\pi\frac{h}{4}\Delta h}$$

- b. [5 points] Assuming the bowl is filled with punch up to a height of  $h = 1/2$ , write an integral which gives the amount of work needed to lift all of the punch over the rim of the bowl. Do not evaluate your integral.

$$\boxed{\text{Solution: } \int_0^{1/2} 1200(9.8)\pi\frac{h}{4}(1-h) dh}$$