2. [10 points] Consider an outdoor pool initially filled with 20,000 gallons of water. Each day 4% of the water in the pool evaporates. Each morning at 10:00am, W gallons of water are added back to the pool where W is a constant.

a. [3 points] Let A_n be the number of gallons of water in the pool immediately after water is added back to the pool for the n^{th} time. Given that $A_1 = 19200 + W$, find A_2 and A_3. Put your final answers in the answer blanks.

Solution:
$A_2 = (20,000)(\frac{24}{25})^2 + W(\frac{24}{25}) + W$.
$A_3 = (20,000)(\frac{24}{25})^3 + W(\frac{24}{25})^2 + W(\frac{24}{25}) + W$.

b. [4 points] Find a closed form expression for A_n (i.e. evaluate any sums and solve any recursion). Note your answer may contain the constant W.

Solution: $A_n = \frac{24}{25}A_{n-1} + W$. Expanding this recursion or following the pattern from part a we have $A_n = 20,000(\frac{24}{25})^n + \sum_{k=0}^{n-1} W(\frac{24}{25})^k$. Using the formula for finite geometric series we have $A_n = 20,000(\frac{24}{25})^n + 25W(1 - (\frac{24}{25})^n)$.

c. [3 points] If the pool has a maximum capacity of 25,000 gallons, find the largest value of W so that the pool does not overflow eventually.

Solution: Depending on the value of W, A_n is always increasing or always decreasing. Therefore the amount of water in the pool is the largest either when it is first filled at 20,000 gallons or when n approaches infinity where we have $\lim_{n\to\infty} A_n = 25W$. Therefore our only restriction is $25W \leq 25,000$ thus $W \leq 1,000$. So the largest possible value is $W = 1,000$.