4. [10 points] Determine whether the following series converge or diverge. Show all of your work and justify your answer.

a. [5 points] \[ \sum_{n=1}^{\infty} \frac{8^n + 10^n}{9^n} \]

Solution: \[ \lim_{n \to \infty} \frac{8^n + 10^n}{9^n} = \infty \] therefore by the \( n^{th} \) term test the series diverges.

b. [5 points] \[ \sum_{n=4}^{\infty} \frac{1}{n^3 + n^2 \cos(n)} \]

Solution: \[ \sum_{n=4}^{\infty} \frac{1}{n^3 + n^2 \cos(n)} \leq \sum_{n=4}^{\infty} \frac{1}{n^2(n - 1)} \leq \sum_{n=4}^{\infty} \frac{1}{n^2}. \] The final series is a convergent \( p \) series since \( p = 2 > 1 \). Therefore the original series converges by comparison.