6. [8 points]

a. [2 points] Find all values of \( p \) for which the integral 
\[
\int_{-\infty}^{\infty} \frac{1}{(x^2 + 4)^p} \, dx
\]
converges. You do not need to show your work. Circle your final answer.

\[
\text{Solution: } \int_{-\infty}^{\infty} \frac{1}{(x^2 + 4)^p} \, dx = \int_{0}^{\infty} \frac{2}{(x^2 + 4)^p} \, dx \leq \int_{1}^{\infty} \frac{2}{x^{2p}} \, dx \text{ thus } p > 1/2.
\]

b. [2 points] Find all values of \( p \) for which the integral 
\[
\int_{e}^{\infty} \frac{e^{px}}{x^3} \, dx
\]
converges. You do not need to show your work. Circle your final answer.

\[
\text{Solution: } \text{If } p \leq 0 \text{ then } \int_{e}^{\infty} \frac{e^{px}}{x^3} \, dx \leq \int_{e}^{\infty} \frac{1}{x^3} \, dx \text{ which converges thus the integral converges if } p \leq 0. \text{ If } p > 0 \text{ then } \lim_{x \to \infty} \frac{e^{px}}{x^3} = \infty \text{ so the integral will diverge. Therefore the answer is } p \leq 0.
\]

c. [4 points] Find the radius of convergence of the Taylor series 
\[
\sum_{n=1}^{\infty} \frac{x^{2n}}{n2^n}.
\]

\[
\text{Solution: Using the ratio test we consider } \lim_{n \to \infty} \left| \frac{x^{2(n+1)}}{x^{2n}} \right| = \lim_{n \to \infty} \left| \frac{x^2}{2(n+1)} \right| = |x^2/2|. \text{ In order for the series to converge we must have } |x^2/2| < 1. \text{ Therefore } |x| < \sqrt{2}. \text{ So the radius of converge is } R = \sqrt{2}.
\]