6. [8 points]

a. [2 points] Find all values of p for which the integral $\int_{-\infty}^{\infty} \frac{1}{(x^2+4)^p}\, dx$ converges. You do not need to show your work. Circle your final answer.

Solution: $\int_{-\infty}^{\infty} \frac{1}{(x^2+4)^p}\, dx = \int_{0}^{\infty} \frac{2}{(x^2+4)^p}\, dx \leq \int_{1}^{\infty} \frac{2}{x^{2p}}/, dx$ thus $p > 1/2$.

b. [2 points] Find all values of p for which the integral $\int_{e}^{\infty} \frac{e^{px}}{x^3}\, dx$ converges. You do not need to show your work. Circle your final answer.

Solution:
If $p \leq 0$ then $\int_{e}^{\infty} \frac{e^{px}}{x^3}\, dx \leq \int_{e}^{\infty} \frac{1}{x^3}\, dx$ which converges thus the integral converges if $p \leq 0$. If $p > 0$ then $\lim_{x\to\infty} frace^{px}x^3 = \infty$ so the integral will diverge. Therefore the answer is $p \leq 0$.

c. [4 points] Find the **radius** of convergence of the Taylor series $\sum_{n=1}^{\infty} \frac{x^{2n}}{n2^n}$.

Solution: Using the ratio test we consider $\lim_{n\to\infty} |\frac{x^{2(n+1)}n2^n}{x^{2n}(n+1)2^{n+1}}| = \lim_{n\to\infty} |\frac{x^2n}{2(n+1)}| = |x^2/2|$. In order for the series to converge we must have $|x^2/2| < 1$. Therefore $|x| < \sqrt{2}$. So the radius of converge is $R = \sqrt{2}$.