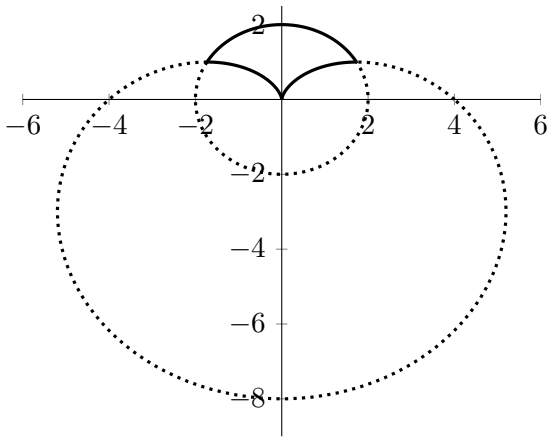
7. [8 points] Roy the alpaca is designing a pool and a deck for his family. The pool has the shape of a cardioid whose equation is given by $r = 4 - 4\sin(\theta)$ where r is in meters and θ is a number between 0 and 2π. The deck will be built in the region that lies inside the circle $x^2 + y^2 = 4$ and outside the cardioid. The deck is depicted in the figure as the region enclosed by the solid lines



a. [1 point] Write the equation for the circle $x^2 + y^2 = 4$ in polar coordinates.

Solution: $r^2 = 4$ so $r = 2$

b. [2 points] Find the values of θ between 0 and 2π where the cardioid and the circle intersect.

Solution: Setting the two equations equal to each other we have $2 = 4 - 4\sin(\theta)$ thus $\sin(\theta) = \frac{1}{2}$. Therefore $\theta = \pi/6, 5\pi/6$.

c. [5 points] Write an expression involving integrals that gives the area of the region where the deck will be built. Do not evaluate your expression.

Solution: $\int_{\pi/6}^{5\pi/6} 2\,d\theta - \int_{\pi/6}^{5\pi/6} \frac{1}{2}(4 - 4\sin(\theta)^2\,d\theta = 4\pi/3 - \int_{\pi/6}^{5\pi/6} \frac{1}{2}(4 - 4\sin(\theta)^2\,d\theta.$