7. [8 points] Roy the alpaca is designing a pool and a deck for his family. The pool has the shape of a cardioid whose equation is given by \( r = 4 - 4 \sin(\theta) \) where \( r \) is in meters and \( \theta \) is a number between 0 and \( 2\pi \). The deck will be built in the region that lies inside the circle \( x^2 + y^2 = 4 \) and outside the cardioid. The deck is depicted in the figure as the region enclosed by the solid lines.

![Cardioid and Circle Diagram](image)

**a. [1 point]** Write the equation for the circle \( x^2 + y^2 = 4 \) in polar coordinates.

\[
\text{Solution: } r^2 = 4 \text{ so } r = 2
\]

**b. [2 points]** Find the values of \( \theta \) between 0 and \( 2\pi \) where the cardioid and the circle intersect.

\[
\text{Solution: Setting the two equations equal to each other we have } 2 = 4 - 4 \sin(\theta) \text{ thus } \sin(\theta) = \frac{1}{2}. \text{ Therefore } \theta = \frac{\pi}{6}, \frac{5\pi}{6}.
\]

**c. [5 points]** Write an expression involving integrals that gives the area of the region where the deck will be built. Do not evaluate your expression.

\[
\text{Solution: } \int_{\pi/6}^{5\pi/6} 2 \, d\theta - \int_{\pi/6}^{5\pi/6} \frac{1}{2} (4 - 4 \sin(\theta))^2 \, d\theta = 4\pi/3 - \int_{\pi/6}^{5\pi/6} \frac{1}{2} (4 - 4 \sin(\theta))^2 \, d\theta.
\]