

4. [9 points] We can define the Bessel function of order one by its Taylor series about $x = 0$,

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}.$$

- a. [3 points] Compute $J_1^{(2015)}(0)$. Write your answer in exact form and do not try evaluate using a calculator.

$$J_1^{(2015)}(0) = \underline{\hspace{2cm}}$$

- b. [4 points] Find $P_5(x)$, the Taylor polynomial of degree 5 that approximates $J_1(x)$ near $x = 0$.

$$P_5(x) = \underline{\hspace{2cm}}$$

- c. [2 points] Use the Taylor polynomial from the previous part to compute

$$\lim_{x \rightarrow 0} \frac{J_1(x) - \frac{1}{2}x}{x^3}.$$

$$\lim_{x \rightarrow 0} \frac{J_1(x) - \frac{1}{2}x}{x^3} = \underline{\hspace{2cm}}$$