1. [10 points] Show that the following series converges. Also, determine whether the series converges conditionally or converges absolutely. Circle the appropriate answer below. **You must show all your work and indicate any theorems you use to show convergence and to determine the type of convergence**.

$$\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n}$$

$\boxed{\text{CONVERGES CONDITIONALLY}}$ CONVERGES ABSOLUTELY

Solution:
The series we obtain when we take the absolute value of the terms in the series above is $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$. Now consider the integral $\int_2^{\infty} \frac{\ln(x)}{x}dx$. By making a change of variables we see that

$$\begin{aligned}\int_2^{\infty} \frac{\ln(x)}{x}dx &= \lim_{b\to\infty} \int_2^{b} \frac{\ln(x)}{x}dx \\ &= \lim_{b\to\infty} \int_{\ln(2)}^{\ln(b)} u du \\ &= \lim_{b\to\infty} \frac{(\ln(b))^2}{2} - \frac{(\ln(2))^2}{2} \\ &= +\infty\end{aligned}$$

and so the integral above diverges. Thus, the integral test implies that $\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$ diverges.

Since $\frac{\ln(n+1)}{n+1} \leq \frac{\ln(n)}{n}$ and $\lim_{n\to\infty} \frac{\ln(n)}{n} = 0$, we have that $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n}$ converges by the alternating series test.

Altogether we have shown that the series $\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n}$ is conditionally convergent.