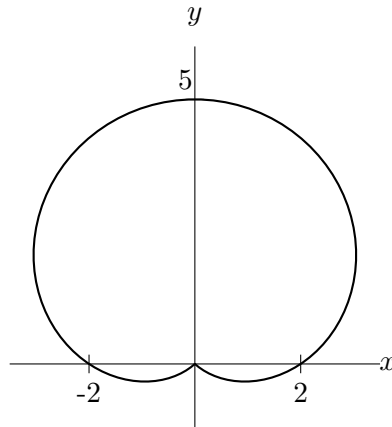


10. [12 points] Vic is watching the ladybugs run around in his garden. His garden is in the shape of the outer loop of a cardioid with polar equation  $r = 2 + 3 \sin \theta$  where  $r$  is measured in meters and  $\theta$  is measured in radians. The outline of the garden is pictured below for your reference. At a time  $t$  minutes after he begins watching, Apple, his favorite red ladybug, is at the  $xy$ -coordinate  $(\sin^2 t, \cos^2 t)$ , and Emerald, his prized green ladybug, is at the  $xy$ -coordinate  $(-\cos(2t), \sin(2t) + 1.5)$ . Vic watches the ladybugs for  $2\pi$  minutes.



Using the information above, circle the correct answer for each part below. There is only one correct answer for each part. You do not need to show your work.

- a. [3 points] Which of the following integrals gives the area of the garden?

*Solution:*

A)  $\frac{1}{2} \int_0^\pi (2 + 3 \sin \theta)^2 d\theta$

B)  $\frac{1}{2} \int_{\arcsin(\frac{2}{3})}^{\pi + \arcsin(\frac{2}{3})} (2 + 3 \sin \theta)^2 d\theta$

C)  $\frac{1}{2} \int_{-\arcsin(\frac{2}{3})}^{\pi + \arcsin(\frac{2}{3})} (2 + 3 \sin \theta)^2 d\theta$

D)  $\frac{1}{2} \int_0^{2\pi} (2 + 3 \sin \theta)^2 d\theta$

E)  $\frac{1}{2} \int_{2\pi - \arcsin(\frac{2}{3})}^{4\pi - \arcsin(\frac{2}{3})} (2 + 3 \sin \theta)^2 d\theta$

- b. [3 points] Which of the following is **not** true about Apple while Vic is watching?

*Solution:*

A) Apple runs through the point  $(\frac{1}{2}, \frac{1}{2})$  more than once.

B) Apple crosses the path made by Emerald exactly 4 times.

C) Apple's speed is zero at least once.

D) Apple does not leave the garden.

E) Apple is moving faster than Emerald for some of the time.

- c. [3 points] How far does Emerald run while Vic is watching?

*Solution:*

A)  $\pi$  meters

B)  $2\pi$  meters

C)  $4\pi$  meters

D)  $\sqrt{8}\pi$  meters

E)  $8\pi$  meters

- d. [3 points] After the  $2\pi$  minutes, Vic stops watching. Apple runs from the point  $(x, y) = (0, 1)$  in the positive  $y$ -direction with speed of  $g(T) = 5Te^{-T}$ ,  $T$  seconds after Vic stops watching. Which of the following is true?

*Solution:*

A) Apple leaves the garden eventually, but never runs further than 5 meters total.

B) Apple's speed is always increasing after Vic stops watching.

C) If given enough time, Apple would eventually be more than 1000 meters from the garden.

D) Apple is still in the garden 5 minutes after Vic stops watching.

E) Apple changes direction, eventually.