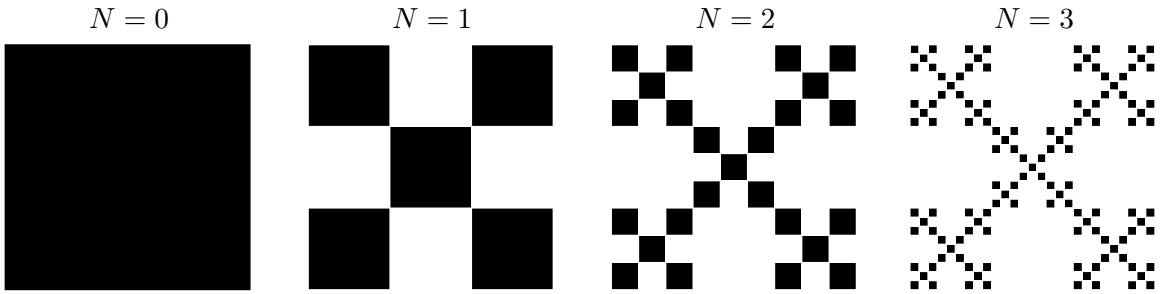
11. [12 points] You construct a snowflake by starting with a square piece of paper of side length 3 inches. You divide the square into a three by three grid of squares of side length one and remove the four squares in the grid that share a side with the center square in the grid. For each remaining square in the grid, subdivide each of them into 9 equally sized squares and remove the four squares in each of these new grids that share a side with the center square in the grid. You continue in this manner for a long time.



a. [3 points] Write a formula that gives the perimeter, P_N, of the black squares that make up the snowflake after N steps.

Solution: $P_N = 12\left(\frac{5}{3}\right)^N$

b. [2 points] Find $\lim_{N\to\infty} P_N$.

Solution: P_N tends to infinity as $N \to \infty$.

c. [3 points] Suppose $N \geq 1$. Write a sum that gives the area, A_N of all the squares you have **removed** after N steps.

Solution: $\sum_{j=0}^{N-1} 4\left(\frac{5}{9}\right)^j$

d. [2 points] Write a closed form expression for A_N.

Solution: $A_N = 4\dfrac{1-\left(\frac{5}{9}\right)^N}{1-\frac{5}{9}}$

e. [2 points] Find the limit as $N \to \infty$ of your expression in (d).

Solution: $\lim_{N\to\infty} A_N = 9$