11. [12 points] You construct a snowflake by starting with a square piece of paper of side length 3 inches. You divide the square into a three by three grid of squares of side length one and remove the four squares in the grid that share a side with the center square in the grid. For each remaining square in the grid, subdivide each of them into 9 equally sized squares and remove the four squares in each of these new grids that share a side with the center square in the grid. You continue in this manner for a long time.

<table>
<thead>
<tr>
<th>$N = 0$</th>
<th>$N = 1$</th>
<th>$N = 2$</th>
<th>$N = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Image of snowflake stages" /></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. [3 points] Write a formula that gives the perimeter, $P_N$, of the black squares that make up the snowflake after $N$ steps.

$$\text{Solution: } P_N = 12 \left( \frac{5}{3} \right)^N$$

b. [2 points] Find $\lim_{N \to \infty} P_N$.

$$\text{Solution: } P_N \text{ tends to infinity as } N \to \infty.$$  

c. [3 points] Suppose $N \geq 1$. Write a sum that gives the area, $A_N$ of all the squares you have removed after $N$ steps.

$$\text{Solution: } \sum_{j=0}^{N-1} 4 \left( \frac{5}{9} \right)^j$$

d. [2 points] Write a closed form expression for $A_N$.

$$\text{Solution: } A_N = 4 \frac{1 - \left( \frac{5}{9} \right)^N}{1 - \frac{5}{9}}$$

e. [2 points] Find the limit as $N \to \infty$ of your expression in (d).

$$\text{Solution: } \lim_{N \to \infty} A_N = 9$$