2. [4 points]

a. [2 points] You are given that the power series $\sum_{n=0}^{\infty} C_n(x+3)^n$ converges when $x=-6$ and diverges when $x=1$. What are the largest and smallest possible values for the radius of convergence R?

Solution:

$$\underline{3} \le R \le \underline{4}$$

b. [2 points] Give the exact value of the infinite series

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}3^{n+1}}{n!} = -3+9-\frac{27}{2}+\frac{81}{6}-\frac{243}{24}+\cdots$$

Solution: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}3^{n+1}}{n!} = -3e^{-3}$ using the Taylor series for xe^x centered at $x=0$.

3. [5 points] Determine whether the following integral converges or diverges. If the integral converges, circle the word "converges". If the integral diverges, circle "diverges". In either case, **you must show all your work and indicate any theorems you use**.

$$\int_0^1 \frac{\cos(x)}{x^2}\,dx$$

CONVERGES $\boxed{\text{DIVERGES}}$

Solution: For $0 \le x \le 1$, we have that $\cos(x) \ge \cos(1)$. We also know that $\cos(1)\displaystyle\int_0^1 \frac{1}{x^2}dx$ diverges. Therefore, $\displaystyle\int_0^1 \frac{\cos(x)}{x^2}dx$ diverges by the comparison test.