**6**. [9 points]

**a**. [3 points] Find the first three nonzero terms in the Taylor series for $\dfrac{1}{\sqrt{1-x^2}}$ centered at $x = 0$.

*Solution:* We have $(1+y)^{-1/2} = 1 - \frac{1}{2}y + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}y^2 + \cdots$. Substituting $y = -x^2$ gives us $\frac{1}{\sqrt{1-x^2}} = 1 + \frac{1}{2}x^2 + \frac{3}{8}x^4 + \cdots$

**b**. [4 points] Use your answer from part (a) to find the first three nonzero terms in the Taylor series for $\arcsin(2x)$ centered at $x = 0$. Recall that $\frac{d}{dx}\arcsin(x) = \frac{1}{\sqrt{1-x^2}}$.

*Solution:* Integrating the series from part (a) termwise gives us

$$\begin{aligned}\arcsin(x) &= \int 1dx + \int \frac{1}{2}x^2dx + \int \frac{3}{8}x^4dx + \cdots \\ &= C + x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \cdots\end{aligned}$$

Since $\arcsin(0) = 0$, we must have $C = 0$. Then $\arcsin(x) = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \cdots$, and from there a substitution gives us that $\arcsin(2x) = 2x + \frac{4}{3}x^3 + \frac{12}{5}x^5 + \cdots$.

**c**. [2 points] Find the values of $x$ for which the Taylor series from part (b) converges.

*Solution:* The steps we took to get a Taylor series expansion for $\arcsin(x)$ do not change the radius of convergence. So, the Taylor series we found for $\arcsin(x)$ converges for $-1 < x < 1$. Then substituting $2x$ for $x$ gives us that the series which gives our answer to (b) converges for $-1 < 2x < 1$, and so $-\frac{1}{2} < x < \frac{1}{2}$.