7. [9 points] Gwen lifts a bucket of sand straight up from the ground to a height of 10 meters at a constant speed of 0.5 meters per second. The sand is leaking out of the bucket at a rate of $r(t) = \frac{1}{t+1}$ kilograms per second, t seconds after she begins lifting. The bucket and the sand in the bucket together weigh 10 kg when she starts lifting. Recall the gravitational constant is $g = 9.8$ m/s^2.

a. [4 points] Suppose $M(x)$ is the mass of the bucket of sand (in kilograms) when she has lifted it x meters from the ground. Find an expression involving integrals for the work Gwen does lifting the bucket. Your answer can include the function M.

Solution: The work Gwen does lifting the bucket is

$$\int_0^{10} M(x)g\,dx.$$

b. [5 points] Find an expression, possibly involving integrals, for $M(x)$, the mass of the bucket of sand after Gwen has lifted it x meters.

Solution: The mass of the bucket after Gwen has lifted it x meters is

$$M(x) = 10 - \int_0^x \frac{2}{2s+1}\,ds.$$