

8. [10 points] Consider the region A in the xy -plane bounded by $y = 1 - x^4$, the y -axis, and the x -axis in the first quadrant. The area of A is $\frac{4}{5}$.

a. [5 points] Suppose N is any positive whole number. Put the following quantities in order from least to greatest. $\text{MID}(N)$, $\text{TRAP}(N)$, $\text{RIGHT}(N)$, $\text{LEFT}(N)$, and the number $\frac{4}{5}$, where all of the approximations listed are for the integral $\int_0^1 (1 - x^4) dx$.

Solution:

$$\underline{\text{RIGHT}(N)} \leq \underline{\text{TRAP}(N)} \leq \underline{4/5} \leq \underline{\text{MID}(N)} \leq \underline{\text{LEFT}(N)}$$

b. [5 points] Write an expression involving integrals that gives the volume of the solid formed by rotating the region A around the y -axis.

Solution: The volume of the solid formed by rotating the region A about the y -axis is $\int_0^1 2\pi x(1 - x^4) dx$ or equivalently $\int_0^1 \pi(1 - y)^{\frac{1}{2}} dy$