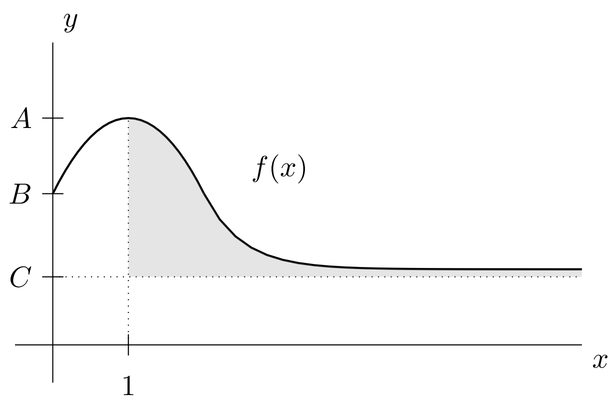
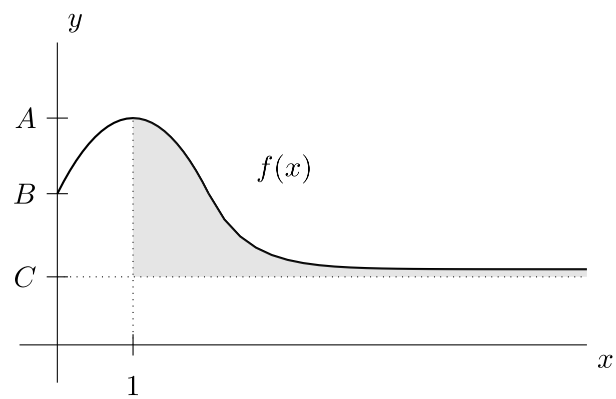
10. [14 points] A function f has domain $[0, \infty)$, and its graph is given below. The numbers A, B, C are positive constants. The shaded region has **finite area**, but it extends infinitely in the positive x-direction. The line $y = C$ is a horizontal asymptote of $f(x)$ and $f(x) > C$ for all $x \geq 0$. The point $(1, A)$ is a local maximum of f.



a. [5 points] Determine the convergence of the improper integral below. **You must give full evidence supporting your answer, showing all your work and indicating any theorems about integrals you use**.

$$\int_0^1 \frac{f(x)}{x}\, dx$$

10. (continued) For your convenience, the graph of f is given again. The numbers A, B, C are positive constants. The shaded region has **finite area**, but it extends infinitely in the positive x-direction. The line $y = C$ is a horizontal asymptote of $f(x)$ and $f(x) > C$ for all $x \geq 0$. The point $(1, A)$ is a local maximum of f.



b. [3 points] **Circle** the correct answer. The value of the integral $\displaystyle\int_1^\infty f(x)f'(x)\,dx$

is $C - A$ is $\frac{C^2-A^2}{2}$ is $B - A$ cannot be determined diverges

c. [3 points] **Circle** the correct answer. The value of the integral $\displaystyle\int_1^\infty f'(x)\,dx$

is $C - A$ is $\frac{C^2-A^2}{2}$ is C cannot be determined diverges

d. [3 points] Determine, with justification, whether the following series converges or diverges.

$$\sum_{n=1}^{\infty}(f(n) - C)$$