3. [12 points] In this problem we study the integral \( I = \int_{1}^{1.5} \ln x \, dx \).

   a. [2 points] Write a left Riemann sum with 5 subdivisions that approximates \( I \), showing all the terms in your sum. **Circle** your sum and leave all the terms in **exact** form.

   \[
   \begin{align*}
   &\frac{1}{5} \left( \ln 1 + \ln 1.1 + \ln 1.2 + \ln 1.3 + \ln 1.4 + \ln 1.5 \right) \\
   &= \frac{1}{5} \left( 0 + \ln 1.1 + \ln 1.2 + \ln 1.3 + \ln 1.4 + \ln 1.5 \right)
   \end{align*}
   \]

   b. [2 points] Use the midpoint rule with 5 subdivisions to approximate \( I \), showing all the terms in your sum. **Circle** your sum and leave all the terms in **exact** form.

   \[
   \begin{align*}
   &\frac{1}{5} \left( \ln 1.05 + \ln 1.15 + \ln 1.25 + \ln 1.35 + \ln 1.45 \right) \\
   &= \frac{1}{5} \left( \ln 1.05 + \ln 1.15 + \ln 1.25 + \ln 1.35 + \ln 1.45 \right)
   \end{align*}
   \]

   c. [4 points] (i) Use the \( u \)-substitution \( u = x - 1 \) to find an integral \( J \), which is equal to \( I \). **Circle** your answer.

   \[
   J = \int_{0}^{0.5} \ln u \, du
   \]

   (ii) Give \( P_3(u) \), the 3rd degree Taylor polynomial around \( u = 0 \) for the integrand of the integral \( J \). **Circle** your answer.

   \[
   P_3(u) = u + \frac{u^2}{2} + \frac{u^3}{3}
   \]

   (iii) Substitute \( P_3(u) \) for the integrand of \( J \) and compute the resulting integral by hand. **Circle** your answer.

   \[
   \int_{0}^{0.5} \left( u + \frac{u^2}{2} + \frac{u^3}{3} \right) \, du
   \]

   \[
   = \left[ \frac{1}{2} u^2 + \frac{1}{4} u^3 + \frac{1}{12} u^4 \right]_{0}^{0.5}
   \]

   \[
   = \frac{1}{2} \cdot 0.5^2 + \frac{1}{4} \cdot 0.5^3 + \frac{1}{12} \cdot 0.5^4
   \]

   \[
   = \frac{1}{8} + \frac{1}{32} + \frac{1}{96}
   \]

   \[
   = \frac{12}{96} + \frac{3}{96} + \frac{1}{96}
   \]

   \[
   = \frac{16}{96}
   \]

   \[
   = \frac{1}{6}
   \]
3. (continued)

d. [4 points] Finally find the exact value of \( I = \int_{1}^{1.5} \ln x \, dx \) using integration by parts. Give your answer in exact form and show your work. Circle your answer.

4. [5 points]

The function \( g(x) \) satisfies the differential equation \( y' = ay^2 - x \). The table on the right gives some information about \( g(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

a. [2 points] Find \( a \).

b. [3 points] Approximate \( g(1.2) \) using Euler’s method with \( \Delta x = 0.1 \).