1. [8 points] Suppose that $f(x)$ is a continuous function, and $F(x)$ is an antiderivative of $f(x)$. Assume that $\int_0^1 F(x)\,dx = 3$. A table of values for $F(x)$ is given below.

x	1	2	3	4	5
$F(x)$	1	-2	-4	3	1

Calculate the following quantities **exactly**. Show your work and do not write any decimal approximations.

a. [2 points] $\int_2^4 f(x)\,dx$

Solution: $\int_2^4 f(x)\,dx = F(4) - F(2) = 3 - (-2) = 5$ by the Fundamental Theorem of Calculus.

b. [2 points] The average value of f over the interval $[3, 5]$.

Solution: $\dfrac{\int_3^5 f(x)\,dx}{5-3} = \dfrac{F(5)-F(3)}{2} = \dfrac{1-(-4)}{2} = \dfrac{5}{2}$

c. [2 points] $\int_0^1 xf(x)\,dx$

Solution: Using integration by parts we have: $\int_0^1 xf(x)\,dx = \big(xF(x)\big)\Big|_0^1 - \int_0^1 F(x)\,dx = F(1) - 0 - 3 = 1 - 3 = -2$

d. [2 points] $\int_0^1 f(2x+1)\,dx$

Solution: Using the u-substitution $u = 2x+1$ we have: $\int_0^1 f(2x+1)\,dx = \frac{1}{2}\int_1^3 f(u)\,du = \frac{1}{2}(F(3) - F(1)) = \dfrac{-4-1}{2} = -\dfrac{5}{2}$