1. [8 points] Suppose that \( f(x) \) is a continuous function, and \( F(x) \) is an antiderivative of \( f(x) \). Assume that \( \int_0^1 F(x) \, dx = 3 \). A table of values for \( F(x) \) is given below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(x) )</td>
<td>1</td>
<td>-2</td>
<td>-4</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Calculate the following quantities \textbf{exactly}. Show your work and do not write any decimal approximations.

a. [2 points] \( \int_2^4 f(x) \, dx \)

Solution: \( \int_2^4 f(x) \, dx = F(4) - F(2) = 3 - (-2) = 5 \) by the Fundamental Theorem of Calculus.

b. [2 points] The average value of \( f \) over the interval \([3, 5] \).

Solution: \( \frac{\int_3^5 f(x) \, dx}{5 - 3} = \frac{F(5) - F(3)}{2} = \frac{1 - (-4)}{2} = \frac{5}{2} \)

c. [2 points] \( \int_0^1 xf(x) \, dx \)

Solution: Using integration by parts we have: \( \int_0^1 xf(x) \, dx = (xF(x)) \bigg|_0^1 - \int_0^1 F(x) \, dx = F(1) - 0 - 3 = 1 - 3 = -2 \)

d. [2 points] \( \int_0^1 f(2x + 1) \, dx \)

Solution: Using the \( u \)-substitution \( u = 2x + 1 \) we have: \( \int_0^1 f(2x+1) \, dx = \frac{1}{2} \int_1^3 f(u) \, du = \frac{1}{2} (F(3) - F(1)) = -\frac{4 - 1}{2} = -\frac{5}{2} \)