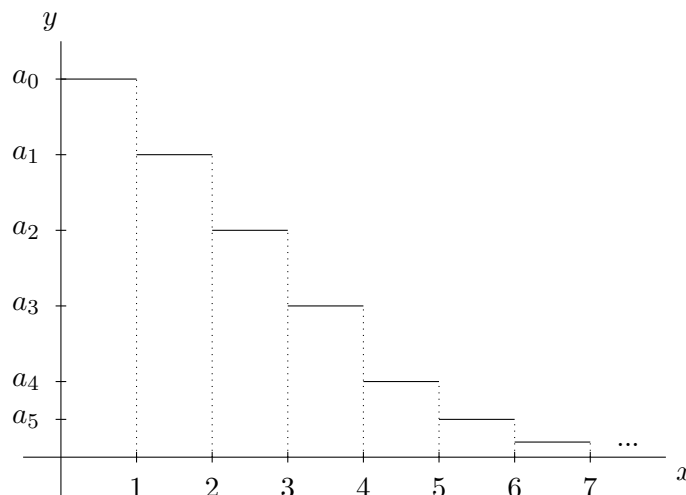


11. [5 points] The Hanoi tower is made by rotating the region depicted below around the  $y$ -axis. The region is made up of infinitely many adjacent rectangles. The  $n$ th rectangle has width 1 and height  $a_n = \frac{1}{n!(2n+1)}$  where  $n = 0, 1, 2, 3, \dots$ . The rectangle touching the  $y$ -axis corresponds to  $n = 0$ . Note that the  $y$ -axis is not to scale.



Compute the volume of the Hanoi Tower. Give an **exact** answer.

*Solution:* To compute the volume of the Hanoi Tower, we focus on each rectangle separately. The volume of the object made by the revolution of the  $n$ th rectangle is given by

$$[\pi(n+1)^2 - \pi n^2] \cdot a_n = \pi(2n+1) \frac{1}{n!(2n+1)} = \frac{\pi}{n!}$$

The total volume is given by adding the volume of all those objects for  $n = 0, 1, 2, 3, \dots$

$$\sum_{n=0}^{\infty} \frac{\pi}{n!} = \pi \cdot \sum_{n=0}^{\infty} \frac{1}{n!} = \pi e$$