3. [12 points] In this problem we study the integral $I = \int_1^{1.5} \ln x \, dx$.

a. [2 points] Write a left Riemann sum with 5 subdivisions that approximates I, showing all the terms in your sum. **Circle** your sum and leave all the terms in **exact** form.

Solution:

$$I \approx LEFT(5) = 0.1 \ln 1 + 0.1 \ln 1.1 + 0.1 \ln 1.2 + 0.1 \ln 1.3 + 0.1 \ln 1.4$$

b. [2 points] Use the midpoint rule with 5 subdivisions to approximate I, showing all the terms in your sum. **Circle** your sum and leave all the terms in **exact** form.

Solution:

$$I \approx MID(5) = 0.1 \ln 1.05 + 0.1 \ln 1.15 + 0.1 \ln 1.25 + 0.1 \ln 1.35 + 0.1 \ln 1.45$$

c. [4 points] (i) Use the u-substitution $u = x - 1$ to find an integral J, which is equal to I. **Circle** your answer.

Solution: $I = \int_0^{0.5} \ln(u+1) \, du = J$

(ii) Give $P_3(u)$, the 3rd degree Taylor polynomial around $u = 0$ for the integrand of the integral J. **Circle** your answer.

Solution: $P_3(u) = u - \frac{u^2}{2} + \frac{u^3}{3}$

(iii) Substitute $P_3(u)$ for the integrand of J and compute the resulting integral by hand. **Circle** your answer.

Solution:

$$J = \int_0^{0.5} \ln(u+1) \, du \approx \int_0^{0.5} u - \frac{u^2}{2} + \frac{u^3}{3} \, du = \left(\frac{u^2}{2} - \frac{u^3}{6} + \frac{u^4}{12} \right) \Big|_0^{0.5} = \frac{1}{8} - \frac{1}{48} + \frac{1}{192} \approx 0.109$$

3. (continued)

d. [4 points] Finally find the exact value of $I = \int_1^{1.5} \ln x \, dx$ using integration by parts. Give your answer in **exact** form and show your work. **Circle** your answer.

Solution: Using integration by parts with $u = \ln x, u' = 1/x, v = x, v' = 1$ we find

$$I = \int_1^{1.5} \ln x \, dx = (x \ln x)\Big|_1^{1.5} - \int_1^{1.5} x \frac{1}{x} \, dx = 1.5 \ln 1.5 - (1.5 - 1) = 1.5 \ln 1.5 - 0.5$$

4. [5 points]

The function $g(x)$ satisfies the differential equation $y' = ay^2 - x$. The table on the right gives some information about $g(x)$.

x	$g(x)$	$g'(x)$
1	1	2

a. [2 points] Find a.

Solution: Plugging the data in the differential equation, we get $2 = a \cdot 1^2 - 1$ which gives $a = 3$.

b. [3 points] Approximate $g(1.2)$ using Euler's method with $\Delta x = 0.1$.

Solution: We use the formula $\Delta y = \Delta x \cdot y'$. We calculate y' from the differential equation $y' = 3y^2 - x$.

x	y	y'
1	1	2
1.1	1.2	3.22
1.2	1.522	...

Thus, $g(1.2) \approx 1.522$.