6. [4 points] Determine the **exact** value of the infinite series in each of the following questions. No decimal approximations are allowed. You do not need to show your work. **Circle** your answer.

a. [2 points] $\frac{1}{5^2} - \frac{1}{5^4} + \frac{1}{5^6} - \frac{1}{5^8} + \frac{1}{5^{10}} - \frac{1}{5^{12}} + \cdots = \frac{\frac{1}{5^2}}{1-(-\frac{1}{5^2})} = \frac{1}{26}$

b. [2 points] $\sum_{n=0}^{\infty} \frac{(-1)^n 5^{2n}}{(2n+1)!} = \frac{1}{5}\sin(5)$

7. [6 points] Consider the differential equation $y' = 1 - 2xy$.

a. [4 points] Suppose k is an arbitrary constant. Show that the function

$$y(x) = \frac{k + \int_2^x e^{t^2}\,dt}{e^{x^2}}$$

is a solution to the differential equation.

Solution: We want to show that $y'(x) = 1 - 2xy(x)$. We show this by showing that both sides are equal to the same quantity:

$$y'(x) = \frac{e^{x^2}e^{x^2} - (k + \int_2^x e^{t^2}\,dt)\cdot e^{x^2}2x}{e^{2x^2}} = 1 - \frac{(k + \int_2^x e^{t^2}\,dt)\cdot 2x}{e^{x^2}}$$

where in the first equality we used both the 2nd Fundamental Theorem and the quotient rule.
On the other hand,

$$1 - 2xy(x) = 1 - 2x\frac{k + \int_2^x e^{t^2}\,dt}{e^{x^2}}.$$

Thus, $y(x)$ satisfies the differential equation.

b. [2 points] Give the value of k so that the graph of the solution to the differential equation passes through the point $(2, 7)$.

Solution: We have

$$y(2) = 7$$
$$\frac{k}{e^4} = 7$$
$$k = 7e^4.$$