10. [8 points] The Taylor series centered at $x = 0$ for a function $F(x)$ converges to $F(x)$ for all x and is given below.

$$F(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n)!(4n+1)}$$

a. [3 points] What is the value of $F^{(101)}(0)$?
Make sure your answer is exact. You do not need to simplify.

$F(x) = \sum_{n=0}^{\infty} \frac{F^{(n)}(0)}{n!} x^n$ by Taylor's Theorem. So

The x^{101} term is $\frac{F^{(101)}(0)}{101!} x^{101}$. When you plug $n=25$ into the series above you get $-\frac{x^{101}}{50!(101)}$

So $F^{(101)}(0) = \frac{-101!}{50!(101)}$

Answer: $F^{(101)}(0) = \boxed{-\frac{100!}{50!}}$

b. [3 points] Find $P_9(x)$, the 9th degree Taylor polynomial that approximates $F(x)$ near $x = 0$.

$$P_9(x) = \frac{x^{0+1}}{0!(0+1)} - \frac{x^{4+1}}{2!(4+1)} + \frac{x^{8+1}}{4!(8+1)}$$

$$= \boxed{x - \frac{1}{10}x^5 + \frac{1}{216}x^9}$$

c. [2 points] Use your Taylor polynomial from part **b.** to compute

$$\lim_{x \to 0} \frac{F(x) - x}{2x^5}$$

$$\lim_{x \to 0} \frac{F(x) - x}{2x^5} = \lim_{x \to 0} \frac{\left[\cancel{x} - \frac{1}{10}x^5 + \frac{1}{216}x^9\right] - \cancel{x}}{2x^5}$$

$$= \lim_{x \to 0} -\frac{1}{20} + \frac{1}{432}x^4 = \boxed{-\frac{1}{20}}$$