11. [12 points] Quinn is a patient taking a new experimental medicine.

a. [4 points] Quinn knows that the amount of the medicine in her body decays at a rate proportional to the current amount of the medicine in her body with constant of proportionality \( k > 0 \). Let \( Q = Q(t) \) be the quantity, in mg, of this medicine that is in Quinn’s body \( t \) days after she begins taking it. Assuming the medicine enters her body at a continuous rate of 200mg per day, write a differential equation that models \( Q(t) \), and give an appropriate initial condition.

**Answer:**

Differential Equation: \[ \frac{dQ}{dt} = 200 - kQ \]

Initial Condition: \( Q(0) = 0 \)

b. [2 points] Find the values of \( Q_1 \), \( Q_2 \) and \( Q_3 \).

**Answers:** \( Q_1 = 200 \) \( Q_2 = \frac{200}{1 - \frac{1}{4}} = \frac{200}{3} \) \( Q_3 = \frac{200}{3} \left[ 1 - \left(\frac{1}{4}\right)^3 \right] \)

c. [4 points] Write a closed form expression for \( Q_n \). (Your answer should not include sigma notation or ellipses (⋯).)

**Answer:** \( Q_n = \frac{200}{3} \left[ 1 - \left(\frac{1}{4}\right)^n \right] \)

d. [2 points] What is \( \lim_{n \to \infty} Q_n \)? Interpret your answer in the context of the problem.

\[ \lim_{n \to \infty} Q_n = \frac{800}{3} \]  
In the long run, the amount of drug in the body just after taking a pill is about 266 mg.