11. [12 points] Quinn is a patient taking a new experimental medicine.

a. [4 points] Quinn knows that the amount of the medicine in her body decays at a rate proportional to the current amount of the medicine in her body with constant of proportionality $k > 0$. Let $Q = Q(t)$ be the quantity, in mg, of this medicine that is in Quinn's body t days after she begins taking it. Assuming the medicine enters her body at a continuous rate of 200mg per day, write a differential equation that models $Q(t)$, and give an appropriate initial condition.

Answer: Differential Equation: $\frac{dQ}{dt} = 200 - kQ$

Initial Condition: $Q(0) = 0$

For parts **b.-d.** below, suppose that the medicine has a half-life of 12 hours in her body and that, rather than entering her body continuously throughout the day, Quinn takes one 200mg pill each morning at 8am.

Let Q_n be the quantity, in mg, of this medicine that is in Quinn's body immediately after she takes the nth pill. For example, Q_1 is the amount of medicine in her body immediately after she takes her first dose.

b. [2 points] Find the values of Q_1, Q_2 and Q_3.

half-life = 12 hrs, so $\frac{1}{4}$ left after 24 hrs

Answers: $Q_1 =$ 200 $\quad Q_2 =$ $200 + \frac{1}{4}(200)$ $\quad Q_3 =$ $200 + \frac{1}{4}(200) + \frac{1}{16}(200)$

c. [4 points] Write a closed form expression for Q_n. (Your answer should not include sigma notation or ellipses $(\cdots)$.)

$$Q_n = 200 + \frac{1}{4}(200) + \left(\frac{1}{4}\right)^2(200) + \cdots + \left(\frac{1}{4}\right)^{n-1}(200)$$

$$= 200\left[1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \cdots + \left(\frac{1}{4}\right)^{n-1}\right]$$

$$= 200\,\frac{1-\left(\frac{1}{4}\right)^n}{1-\frac{1}{4}}$$

Answer: $Q_n =$ $\frac{800}{3}\left[1 - \left(\frac{1}{4}\right)^n\right]$

d. [2 points] What is $\lim_{n\to\infty} Q_n$? Interpret your answer in the context of the problem.

$\lim_{n\to\infty} Q_n = \frac{800}{3}$. In the long run, the amount of drug in the body just after taking a pill is about 266 mg.