12. [12 points] For each of the statements and questions below, circle all of the available choices that correctly complete the statement or answer the question. Circle “NONE OF THESE” if none of the available choices are correct. No justification is required. No credit will be awarded for unclear markings.

a. [3 points] Let \( a_n = \int_0^1 \ln(x) \, dx \). Then the sequence \( a_n \)

- i. diverges
- ii. converges to \(-1\)
- iii. is bounded
- iv. is increasing
- v. NONE OF THESE

b. [3 points] Suppose \( K'(x) = \int e^t \sin(t) \, dt \). Then \( K'(x) \) is equal to

- i. \( e^{\sin(x)} \sin(x) \cos(x) \)
- ii. \( e^{\sin(x)} \sin(x) \cos(x) - e^e \sin(e) \)
- iii. \( e^{\sin(x)} \sin(x) \)

Suppose \( F \) is an antiderivative of \( e^e \sin(e) \).

Then \( K'(x) = \frac{1}{e^e} \int_{0}^{\sin(x)} F'(t) \, dt = \frac{1}{e^e} [F(\sin(x)) - F(0)] = e^{\sin(x)} \sin(x) \cos(x) \).

c. [3 points] The radius of convergence of the Taylor series centered at \( w = 0 \) for the function \( f(w) = (1 + 3w^2)^{1/3} \) is

- i. \( \sqrt{3} \)
- ii. \( \frac{1}{3} \)
- iii. \( \frac{1}{\sqrt{3}} \)
- iv. \( \left( \frac{1}{3} \right)^{1/3} \)
- v. NONE OF THESE

Series for \((1+x)^p\) converges if \(-1 < x < 1\), so need \(-1 < 3w^2 < 1\) or \(-\frac{1}{\sqrt{3}} < w < \frac{1}{\sqrt{3}}\).

d. [3 points] If \( k \) is a constant with \( k > 1 \), for which of the following series does the series definitely converge with sum equal to \( k \)?

- i. \( 1 + \ln(k) + \frac{(\ln(k))^2}{2!} + \frac{(\ln(k))^3}{3!} + \frac{(\ln(k))^4}{4!} + \cdots = e^{\ln(k)} = k \)
- ii. \( 2k - \frac{2k(\pi/3)^2}{2!} + \frac{2k(\pi/3)^4}{4!} - \frac{2k(\pi/3)^6}{6!} + \cdots = 2k \cos \frac{\pi}{3} = k \)
- iii. \( k - 1 + \frac{k - 1}{k} + \frac{k - 1}{k^2} + \frac{k - 1}{k^3} + \cdots = (k-1) \left[ 1 + \frac{1}{k} + \frac{1}{k^2} + \cdots \right] = (k-1) \left[ \frac{k}{k-1} \right] = k \)
- iv. \( k(\pi/2) - \frac{k(\pi/2)^3}{3!} + \frac{k(\pi/2)^5}{5!} - \cdots \)

\( k \sin \left( \frac{\pi}{3} \right) = k \)