12. [12 points] For each of the statements and questions below, circle <u>all</u> of the available choices that correctly complete the statement or answer the question.
Circle "NONE OF THESE" if none of the available choices are correct.
No justification is required. No credit will be awarded for unclear markings.

a. [3 points] Let $a_n = \int_{\frac{1}{n}}^{1} \ln(x)\,dx$. Then the sequence a_n

i. diverges

$\boxed{\text{ii. converges to } -1}$

$\boxed{\text{iii. is bounded}}$

iv. is increasing

v. NONE OF THESE

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left[x\ln x - x\right]_{1/n}^{1} = \lim_{n\to\infty} -1 - \left[\frac{1}{n}\ln\frac{1}{n} - \frac{1}{n}\right]$
$= \lim_{n\to\infty} -1 + \frac{\ln n}{n} = -1$

b. [3 points] Suppose $K(x) = \int_{e}^{\sin(x)} e^t \sin(t)\,dt$. Then $K'(x)$ is equal to

$\boxed{\text{i. } e^{\sin(x)}\sin(\sin(x))\cos(x)}$

ii. $e^{\sin(x)}\sin(\sin(x))\cos(x) - e^e\sin(e)e$

iii. $e^{\sin(x)}\sin(x)$

iv. $e^{\sin(x)}\sin(\sin(x))$

v. NONE OF THESE

Suppose F is an antiderivative of $e^t\sin(t)$.
Then $K'(x) = \frac{d}{dx}\int_e^{\sin x} F'(t)\,dt = \frac{d}{dx}\left[F(\sin x) - F(e)\right]$
$= F'(\sin x)\cos x$
$= e^{\sin x}\sin(\sin x)\cos x$.

c. [3 points] The radius of convergence of the Taylor series centered at $w = 0$ for the function $f(w) = (1+3w^2)^{1/3}$ is

i. $\sqrt{3}$ ii. $\frac{1}{3}$ $\boxed{\text{iii. } \frac{1}{\sqrt{3}}}$ iv. $\left(\frac{1}{3}\right)^{1/3}$ v. NONE OF THESE

Series for $(1+x)^p$ converges if $-1 < x < 1$, so need
$-1 < 3w^2 < 1 \iff -\frac{1}{\sqrt{3}} < w < \frac{1}{\sqrt{3}}$

d. [3 points] If k is a constant with $k > 1$, for which of the following series does the series definitely converge with sum equal to k?

$\boxed{\text{i.}}$ $1 + \ln(k) + \frac{(\ln(k))^2)}{2!} + \frac{(\ln(k))^3}{3!} + \frac{(\ln(k))^4}{4!} + \cdots$ $= e^{\ln k} = k$ ✓

$\boxed{\text{ii.}}$ $2k - \frac{2k(\pi/3)^2}{2!} + \frac{2k(\pi/3)^4}{4!} - \frac{2k(\pi/3)^6}{6!} + \cdots$ $= 2k\cos\frac{\pi}{3} = k$ ✓

$\boxed{\text{iii.}}$ $k - 1 + \frac{k-1}{k} + \frac{k-1}{k^2} + \frac{k-1}{k^3} + \cdots$ $= (k-1)\left[1 + \frac{1}{k} + \frac{1}{k^2} + \cdots\right]$
$= (k-1)\frac{1}{1-\frac{1}{k}} = k$ ✓

$\boxed{\text{iv.}}$ $k(\pi/2) - \frac{k(\pi/2)^3}{3!} + \frac{k(\pi/2)^5}{5!} - \cdots$

v. NONE OF THESE

" $k\sin\left(\frac{\pi}{2}\right) = k$ ✓