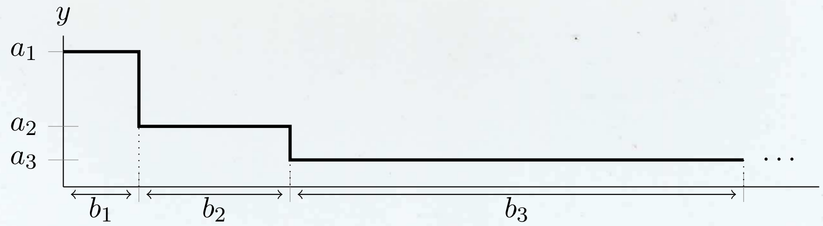
2. [7 points] The region depicted below consists of infinitely many adjacent rectangles. (Only the first three rectangles are actually shown, and they are not necessarily drawn to scale.) For $n = 1, 2, 3, \ldots$, the nth rectangle has height $a_n = \dfrac{1}{5^{n/2}}$ and width $b_n = n!$.



2

a. [5 points] Write an infinite series that gives the total volume of the solid formed by rotating the entire region (all of the rectangles) around the x-axis.

Each step becomes a cylinder:

$$\text{Volume} = \pi a_1^2 b_1 + \pi a_2^2 b_2 + \cdots$$

$$= \pi \sum_{n=1}^{\infty} a_n^2 b_n = \pi \sum_{n=1}^{\infty} \frac{n!}{(5^{n/2})^2}$$

$$= \pi \sum_{n=1}^{\infty} \frac{n!}{5^n}.$$

b. [2 points] Does the infinite series that gives the total volume of the solid formed by rotating the entire region (all of the rectangles) around the x-axis converge or diverge?

Circle one: **Converges** $\boxed{\textbf{Diverges}}$

State the name of the test you would use to justify your answer. If you would use the comparison test or limit comparison also give a valid comparison series. You do not need to actually write out a full justification. (If you do not know the name of the test you would use, state the test itself.)

RATIO TEST:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+1)!/5^{n+1}}{n!/5^n}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \frac{5^n}{5^{n+1}} = \lim_{n\to\infty} \frac{n+1}{5} = \infty.$$

Since the limit is greater than 1, the series diverges.