3. [9 points] In this problem you must give full evidence supporting your answer, showing all your work and indicating any theorems or tests about series you use. (Remark: You **cannot** use any results about convergence from the team homework without justification.)

a. [4 points] Determine whether the series below converges or diverges, and circle your answer clearly. Justify your answer as described above.

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{\sqrt{n}}\right)$$

Converges $\boxed{\textbf{Diverges}}$

Limit compare with $\sum \frac{1}{\sqrt{n}}$:

$$\lim_{n\to\infty} \frac{\sin(1/\sqrt{n})}{1/\sqrt{n}} \overset{LH}{=} \lim_{n\to\infty} \frac{\cos\left(\frac{1}{\sqrt{n}}\right)\cdot \cancel{-\frac{1}{2}(n)^{-3/2}}}{\cancel{-\frac{1}{2}(n)^{-3/2}}}$$

(Top and bottom approach 0 as $n\to\infty$, so can use L'Hôpital's Rule) $= \lim_{n\to\infty} \cos\left(\frac{1}{\sqrt{n}}\right) = \cos(0) = 1.$

$\sum_{1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by the p-test ($p=\frac{1}{2}$), so $\sum \sin\left(\frac{1}{\sqrt{n}}\right)$ diverges by limit comparison.

b. [5 points] Determine if the following infinite series converges absolutely, converges conditionally, or diverges, and circle your answer clearly. Justify your answer as described above.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$$

Converges Absolutely $\boxed{\textbf{Converges Conditionally}}$ **Diverges**

Alternates ✓
|terms| decreasing ✓
terms $\to 0$ ✓
So series converges by the alternating series test.

But: $\sum \left|\frac{(-1)^n}{n\ln(n)}\right| = \sum \frac{1}{n\ln(n)}$ can be resolved using the integral test:

$$\int_2^\infty \frac{dx}{x\ln x} = \lim_{b\to\infty} \ln\ln x \Big|_2^b = \lim_{b\to\infty} \ln\ln b - \ln\ln 2 = \infty.$$

So $\sum$ |terms| diverges by the integral test.