3. [9 points] In this problem you must give full evidence supporting your answer, showing all your work and indicating any theorems or tests about series you use. (Remark: You cannot use any results about convergence from the team homework without justification.)

a. [4 points] Determine whether the series below converges or diverges, and circle your answer clearly. Justify your answer as described above.

\[
\sum_{n=1}^{\infty} \sin \left( \frac{1}{\sqrt{n}} \right)
\]

**Converges**

\[
\lim_{n \to \infty} \frac{\sin \left( \frac{1}{\sqrt{n}} \right)}{\frac{1}{\sqrt{n}}} = \lim_{n \to \infty} \frac{\cos \left( \frac{1}{\sqrt{n}} \right) \cdot \left( -\frac{1}{2} \right) \cdot (n)^{-3/2}}{-\frac{1}{2} (n)^{-3/2}}
\]

(Top and bottom approach 0 as \( n \to \infty \), so can use L'Hôpital's Rule)

\[
\lim_{n \to \infty} \cos \left( \frac{1}{\sqrt{n}} \right) = \cos (0) = 1
\]

\[
\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \text{ diverges by the } p\text{-test } (p = \frac{1}{2}), \text{ so } \sum_{n=1}^{\infty} \sin \left( \frac{1}{\sqrt{n}} \right) \text{ diverges by limit comparison.}
\]

b. [5 points] Determine if the following infinite series converges absolutely, converges conditionally, or diverges, and circle your answer clearly. Justify your answer as described above.

\[
\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(n)}
\]

**Converges Absolutely**

Alternates \( \checkmark \)

Terms decreasing \( \checkmark \)

Terms \( \to \) 0

So series converges by the alternating series test.

**Converges Conditionally**

But: \( \sum \left| \frac{(-1)^n}{n \ln(n)} \right| = \sum \frac{1}{n \ln(n)} \)

Can be resolved using the integral test:

\[
\int_{2}^{\infty} \frac{dx}{x \ln(x)} = \lim_{b \to \infty} \left[ \ln \ln x \right]_{2}^{b} = \lim_{b \to \infty} \ln \ln b - \ln \ln 2 = \infty.
\]

So \( \sum \left| \text{terms} \right| \text{ diverges by the integral test.} \)