4. [7 points] The position of a particle in the plane is given by a pair of parametric equations $x = x(t)$ and $y = y(t)$ where x and y are measured in meters and t is measured in seconds. The functions $x(t)$ and $y(t)$ satisfy the differential equations

$$\frac{dx}{dt} = p(x) \qquad \text{and} \qquad \frac{dy}{dt} = h(t)$$

for functions $p(x)$ and $h(t)$. Some values of the functions p and h are provided in the tables below.

x	0	1	2	3	4	5
$p(x)$	1	4	6	-2	0	3

t	0	0.5	1	1.5	2
$h(t)$	2	-4	1	0	3

a. [4 points] Suppose that $x(0) = 3$ and $y(0) = 2$. Use Euler's method with $\Delta t = 0.5$ to approximate the values of $x(1)$ and $y(1)$. Show your calculation for each step of Euler's method.

t	x	$dx/dt = p(x)$	Δt	$\Delta x = \left(\frac{dx}{dt}\right)\Delta t$
0	3	-2	.5	-1
.5	2	6	.5	3
1	$\boxed{5}$			

t	y	$dy/dt = h(t)$	Δt	$\Delta y = \left(\frac{dy}{dt}\right)\Delta t$
0	2	2	.5	1
.5	3	-4	.5	-2
1	$\boxed{1}$			

Answer: $x(1) \approx$ 5

$y(1) \approx$ 1

b. [3 points] Suppose that $x(2) = 1$ and $y(2) = 5$. How fast is the particle moving when $t = 2$?

$$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{p(x)^2 + h(t)^2}$$

$$= \sqrt{p(1)^2 + h(2)^2} = \sqrt{4^2 + 3^2} = \boxed{5}$$