4. [7 points] The position of a particle in the plane is given by a pair of parametric equations $x = x(t)$ and $y = y(t)$ where $x$ and $y$ are measured in meters and $t$ is measured in seconds. The functions $x(t)$ and $y(t)$ satisfy the differential equations

$$\frac{dx}{dt} = p(x) \quad \text{and} \quad \frac{dy}{dt} = h(t)$$

for functions $p(x)$ and $h(t)$. Some values of the functions $p$ and $h$ are provided in the tables below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>-2</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(t)$</td>
<td>2</td>
<td>-4</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

a. [4 points] Suppose that $x(0) = 3$ and $y(0) = 2$. Use Euler’s method with $\Delta t = 0.5$ to approximate the values of $x(1)$ and $y(1)$. Show your calculation for each step of Euler’s method.

\[
\begin{array}{c|c|c|c|c}
 t & x & \Delta t & p(x) & \Delta x = (\frac{dx}{dt})\Delta t \\
 0 & 3 & & 4 & \\
 0.5 & 2 & 6 & -2 & 1 \\
 1 & 5 & 3 & 0 & 3 \\
\end{array}
\]

| $t$ | 0 & 0.5 & 1 & 1.5 & 2 |
|-----|----------|----------|----------|----------|----------|
| $y$ | 2 & -4 & 1 & 0 & 3 |

\[
\begin{array}{c|c|c|c|c}
 t & y & \Delta t & h(t) & \Delta y = (\frac{dy}{dt})\Delta t \\
 0 & 2 & & 2 & \\
 0.5 & 1 & 1 & -1 & 2 \\
 1 & 3 & 3 & 1 & -2 \\
\end{array}
\]

\[
\text{Answer: } x(1) \approx \boxed{5} \\
y(1) \approx \boxed{1}
\]

b. [3 points] Suppose that $x(2) = 1$ and $y(2) = 5$. How fast is the particle moving when $t = 2$?

\[
\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{p(x)^2 + h(t)^2}
\]

\[
= \sqrt{p(1)^2 + h(2)^2} = \sqrt{4^2 + 5^2} = \boxed{5}
\]