5. [10 points] For each of the problems below, fill in the letter (A-I) corresponding to the answer from the list of answer choices below which correctly finishes the sentence. If none of those options is correct, choose "NONE OF THESE". Note that answer choices may be used more than once.

Answer Choices:

A. equals 0
B. equals 1
C. equals 2
D. equals 3
E. equals 4
F. equals 5
G. equals 6
H. equals 7
I. diverges
J. NONE OF THESE

a. [2 points] The integral $\int_0^\infty 5xe^{-x}\,dx$

Let $u = 5x$ $\quad v' = e^{-x}$
$u' = 5$ $\quad v = -e^{-x}$

$$= \int_0^\infty uv' = uv\Big|_0^\infty - \int_0^\infty u'v = \lim_{b\to\infty} -\frac{5x}{e^x}\Big|_0^b - \int_0^b -5e^{-x}$$

$$= \lim_{b\to\infty}\left[\frac{-5x}{e^x} - \frac{5}{e^x}\right]_0^b = 5$$

Answer: $\boxed{F}$

b. [2 points] The series $(e-1) - \frac{(e-1)^2}{2} + \frac{(e-1)^3}{3} + \cdots + (-1)^{n-1}\frac{(e-1)^n}{n} + \cdots$

This is the series for $\ln(1+x)$, evaluated at $x = e-1$. But, the radius of convergence of the series is 1, and $e-1 > 1$. So diverges.

Answer: $\boxed{I}$

c. [2 points] The arclength of the polar curve $r = \frac{\sin(\theta)}{\pi}$

Curves of the form $r = a\sin\theta$ are circles of radius a centered at $(0,a)$, and they complete a revolution from $\theta = 0$ to $\theta = \pi$. So

length $= \int_0^\pi \sqrt{f(\theta)^2 + f'(\theta)^2}\,d\theta$

$$= \int_0^\pi \sqrt{\frac{1}{\pi^2}\sin^2\theta + \frac{1}{\pi^2}\cos^2\theta} = \int_0^\pi \frac{d\theta}{\pi} = 1.$$

Answer: $\boxed{B}$

d. [2 points] The TRAP(2) estimate for the integral $\int_0^1 8x^2\,dx$

$$\frac{1}{2}\Delta x\left[y_0 + 2y_1 + y_2\right]$$
$$= \frac{1}{2}\left(\frac{1}{2}\right)\left[f(0) + 2f\left(\tfrac{1}{2}\right) + f(1)\right]$$
$$= \frac{1}{4}\left[8(0)^2 + 2\cdot 8(.5)^2 + 8(1)^2\right]$$
$$= \frac{1}{4}\cdot 8\left[0 + 2\left(\tfrac{1}{4}\right) + 1\right] = 3$$

Answer: $\boxed{D}$

e. [2 points] The series $\sum_{n=3}^{\infty}\left(\frac{12}{n} - \frac{12}{n+2}\right)$

$$= 12\Big[\ 1/3 + 1/4 + \cancel{1/5} + \cancel{1/6} + \cdots$$
$$- \cancel{1/5} - \cancel{1/6} - \cancel{1/7} - \cancel{1/8} - \cdots\Big] = 12\left[\frac{1}{3} + \frac{1}{4}\right] = 7$$

Answer: $\boxed{H}$