8. [8 points] A very unique sand dollar has an interesting pattern on it. The outline of the sand dollar is given by the polar equation \( r = 2 \). On the face of the sand dollar are many other polar curves \( c_n \). For \( n = 1, 2, 3, \ldots \), the curve \( c_n \) is given by the polar equation \( r = \frac{1}{n} \sin(5\theta) \). Below is a picture of the sand dollar (left) and a zoomed in view of two of the polar curves (right).

   ![Polar curves](image)

   a. [3 points] Let \( a_0 \) be the area that is inside the sand dollar but outside \( c_1 \). Write a formula involving one or more integrals for \( a_0 \).
   
   \[
   a_0 = 4\pi - 5 \left( \text{area of one petal of } c_1 \right) = 4\pi - 5 \cdot \frac{1}{2} \int_0^{\pi/5} \sin^2(5\theta) \, d\theta
   \]

   b. [3 points] For \( n \geq 1 \), let \( a_n \) be the area inside \( c_n \) but outside \( c_{n+1} \). Write a formula involving one or more integrals for \( a_n \) when \( n \geq 1 \).
   
   \[
   a_n = \frac{5}{2} \int_0^{\pi/5} \frac{1}{n^2} \sin^2(5\theta) \, d\theta - \frac{5}{2} \int_0^{\pi/5} \frac{1}{(n+1)^2} \sin^2(5\theta) \, d\theta
   \]

   c. [2 points] Does the infinite series \( \sum_{n=0}^{\infty} a_n \) converge or diverge? If it converges, what is its value? No justification is necessary.

   \[ a_0 + a_1 + a_2 + \ldots \text{ converges to the area of the whole sand dollar, which is } 4\pi. \]