9. [8 points] This problem concerns a rocket that has been launched and is ascending. You may assume the acceleration due to gravity is $g = 9.8$ m/s^2.

Because it is burning fuel, the rocket's mass is decreasing. Let $m(h)$ be the mass (in kg) of the rocket during its ascent when it is at a height of h meters above the ground.

a. [2 points] Suppose Δh is small. Write an expression (not involving integrals) in terms of m and h that approximates the work (in joules) required for the rocket to ascend from a height of h meters above the ground to a height of $h + \Delta h$ meters above the ground.

mass at height h = $m(h)$
weight " " " = $m(h)g$
dist to lift rocket = Δh
$\therefore$ work " " " = $m(h)g\Delta h$ = **$9.8\, m(h)\Delta h$ joules**

b. [2 points] Write, but do **not** evaluate, an integral that gives the total work (in joules) required for the rocket to ascend from a height of 100 meters above the ground to a height of 2500 meters above the ground.

$$\int_{100}^{2500} 9.8\, m(h)\, dh$$

Let $v(h)$ be the rocket's velocity (in m/s) when it is at a height of h meters above the ground.

c. [2 points] Suppose Δh is small. Write an expression (not involving integrals) in terms of v and h that approximates the time (in seconds) it takes for the rocket to ascend from a height of h meters above the ground to a height of $h + \Delta h$ meters above the ground.

$$v(h) = \frac{dh}{dt} \approx \frac{\Delta h}{\Delta t} \quad \text{so} \quad \Delta t \approx \frac{\Delta h}{v(h)}$$

d. [2 points] Write, but do **not** evaluate, an integral that gives the total time (in seconds) it takes for the rocket to ascend from a height of 100 meters above the ground to a height of 2500 meters above the ground.

$$\int_{100}^{2500} \frac{dh}{v(h)}$$