- 10. [8 points] Recently Debra McQueath was thinking about all the great things she used to make at Print.juice by revolving regions around the *y*-axis. Those were the good days, weren't they?
 - **a**. [4 points]

There was that one time she designed the Juice TitanTM formed by rotating the region in the first quadrant bounded by $x = \pi$ and $y = 4 - x + \cos(4x)$ around the y-axis. The density $\delta(x)$ of the plastic was a function of the distance from the center of the juicer, although Debra cannot quite remember what it was. Help Debra write an integral that represents the total mass of the Juice TitanTM. Your integral may include the density function $\delta(x)$.



Solution: Use the shell method.

Answer:
$$\int_{0}^{\pi} 2\pi x \delta(x) (4 - x + \cos(4x)) dx$$

b. [4 points]

On Debra's last day at Print.juice her team made her a commemorative hat containing a hollow chamber filled with juice by rotating the region bounded by y = 0, y = 1 - x and y = 1 - 2x around the yaxis. The juice-filled hat still sits on her kitchen table; she sometimes wonders how much juice is in the hat. Write an integral that represents the total volume of juice in the hat. **Note:** juice fills the solid formed by rotating the shaded region.



Solution: Use the washer method.

Answer:
$$\int_0^1 \pi ((1-y)^2 - ((1-y)/2)^2) dy$$