2. [9 points] For a class project, Yennifer is studying the accumulation of dead leaves on the ground in a particular region on Nichols Arboretum.

  a. [4 points] She finds that the dead leaves accumulate at a constant rate of 6 grams per square centimeter per year. At the same time, the leaves on the ground decompose at a continuous rate of 80 percent per year. Write a differential equation for the total quantity Q of dead leaves, in grams per square centimeter, at time t, in years.

Answer: $\dfrac{dQ}{dt} = 6 - .8Q$

  b. [5 points] Yennifer finds that if she covers the ground in purified muck, then the total quantity P, in grams, of dead leaves per square centimeter satisfies the differential equation

$$\frac{dP}{dt} = (P - 3)\cos(2\pi t)$$

In addition, when she first applies the muck (at $t = 0$), the ground is covered with 1 gram per square centimeter of leaves.

Use separation of variables to find a formula for $P(t)$. Show your work.

Solution: Separating variables gives

$$\int \frac{1}{P - 3}\, dP = \int \cos(2\pi t)\, dt$$

$$\ln|P - 3| = \frac{1}{2\pi}\sin(2\pi t) + C_0$$

$$P = C_1 e^{\frac{1}{2\pi}\sin(2\pi t)} + 3.$$

Using the initial condition $P(0) = 1$ we find $1 = C_1 + 3$, hence $C_1 = -2$.

Answer: $P(t) =$ $3 - 2e^{\frac{1}{2\pi}\sin(2\pi t)}$